Mathematics

TEXTBOOK FOR CLASS VI

0650

विद्यया ऽमृतमश्नुते

एन सी ई आर टी
NCERT

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OFFICES OF THE PUBLICATION DIVISION, NCERT

NCERT Campus
Sri Aurobindo Marg
New Delhi 110 016 **Phone : 011-26562708**

108, 100 Feet Road
Hosdakere Halli Extension
Banashankari III Stage
Bengaluru 560 085 **Phone : 080-26725740**

Navjivan Trust Building
P.O.Navjivan
Ahmedabad 380 014 **Phone : 079-27541446**

CWC Campus
Opp. Dhankal Bus Stop
Panihati
Kolkata 700 114 **Phone : 033-25530454**

CWC Complex
Maligaon
Guwahati 781 021 **Phone : 0361-2674869**

Publication Team

Head, Publication Division	: *Anup Kumar Rajput*
Chief Production Officer	: *Arun Chitkara*
Chief Business Manager	: *Vipin Dewan*
Chief Editor (In charge)	: *Bijnan Sutar*
Production Assistant	: *Om Prakash*

Cover and layout
Shweta Rao

Illustrations
Anagha Inamdar and *Prashant Soni*

Foreword

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

The National Council of Educational Research and Training (NCERT) appreciates the hard work done by the Textbook Development Committee responsible for this textbook. We wish to thank the Chairperson of the advisory group in Science and Mathematics, Professor J.V. Narlikar and the Chief Advisor for this textbook, Dr. H.K. Dewan for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. We are especially grateful to the members of the National Monitoring Committee, appointed by the Department of Secondary and Higher Education, Ministry of Human Resource Development under the Chairpersonship of Professor Mrinal Miri and Professor G.P. Deshpande, for their valuable time and contribution. As an organisation committed to the systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director

New Delhi
20 November 2006

National Council of Educational
Research and Training

Rationalisation of Content in the Textbooks

In view of the COVID-19 pandemic, it is imperative to reduce content load on students. The National Education Policy 2020, also emphasises reducing the content load and providing opportunities for experiential learning with creative mindset. In this background, the NCERT has undertaken the exercise to rationalise the textbooks across all classes. Learning Outcomes already developed by the NCERT across classes have been taken into consideration in this exercise.

Contents of the textbooks have been rationalised in view of the following:

- Overlapping with similar content included in other subject areas in the same class
- Similar content included in the lower or higher class in the same subject
- Difficulty level
- Content, which is easily accessible to students without much interventions from teachers and can be learned by children through self-learning or peer-learning
- Content, which is irrelevant in the present context

This present edition, is a reformatted version after carrying out the changes given above.

Textbook Development Committee

Chairperson, Advisory Group in Science and Mathematics

J.V. Narlikar, *Emeritus Professor,* Inter University Centre for Astronomy & Astrophysics (IUCCA), Ganeshkhind, Pune University, Pune

Chief Advisor

Dr. H.K. Dewan, Vidya Bhawan Society, Udaipur, Rajasthan

Chief Coordinator

Hukum Singh, *Professor,* DESM, NCERT, New Delhi

Members

Anjali Gupte, *Teacher,* Vidya Bhawan Public School, Udaipur, Rajasthan

Avantika Dam, *TGT,* CIE Experimental Basic School, Department of Education, Delhi

Dharam Prakash, *Reader,* CIET, NCERT, New Delhi

H.C. Pradhan, *Professor,* Homi Bhabha Centre for Science Education, TIFR, Mumbai, Maharashtra

Harsha J. Patadia, *Senior Reader,* Centre of Advance Study in Education, M.S. University of Baroda, Vadodara, Gujarat

Jabashree Ghosh, *TGT,* DM School, RIE, NCERT, Bhubaneswar, Orissa

Mahendra Shankar, *Lecturer (S.G.) (Retd.),* NCERT, New Delhi

Meena Shrimali, *Teacher,* Vidya Bhawan Senior Secondary School, Udaipur, Rajasthan

R. Athmaraman, *Mathematics Education Consultant,* TI Matric Higher Secondary School and AMTI, Chennai, Tamil Nadu

S. Pattanayak, *Professor,* Institute of Mathematics and Application, Bhubaneswar, Orissa

S.K.S. Gautam, *Professor,* DESM, NCERT, New Delhi

Shraddha Agarwal, *PGT,* Sir Padampat Singhania Education Centre, Kanpur, (U.P.)

Srijata Das, *Sr. Lecturer* (Mathematics), SCERT, New Delhi

U.B. Tewari, *Professor,* Department of Mathematics, IIT, Kanpur, (U.P.)

Uaday Singh, *Lecturer,* DESM, NCERT, New Delhi

Member-coordinators

Ashutosh K. Wazalwar, *Professor,* DESM, NCERT, New Delhi

Praveen K. Chaurasia, *Lecturer,* DESM, NCERT, New Delhi

CONSTITUTION OF INDIA

Part IV A (Article 51 A)

Fundamental Duties

Fundamental Duties – It shall be the duty of every citizen of India —

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

(k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

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The book was reviewed by Rahul Sofat, *PGT* Mathematics, Air Force Golden Jubilee School, Subroto Park, New Delhi; Gurpreet Bhatnagar, CBSE Resource Person; A. K. Wazalwar and Til Prasad Sarma, DESM, NCERT, New Delhi.

CONSTITUTION OF INDIA

Part III (Articles 12 – 35)
(Subject to certain conditions, some exceptions
and reasonable restrictions)

guarantees these

Fundamental Rights

Right to Equality
- before law and equal protection of laws;
- irrespective of religion, race, caste, sex or place of birth;
- of opportunity in public employment;
- by abolition of untouchability and titles.

Right to Freedom
- of expression, assembly, association, movement, residence and profession;
- of certain protections in respect of conviction for offences;
- of protection of life and personal liberty;
- of free and compulsory education for children between the age of six and fourteen years;
- of protection against arrest and detention in certain cases.

Right against Exploitation
- for prohibition of traffic in human beings and forced labour;
- for prohibition of employment of children in hazardous jobs.

Right to Freedom of Religion
- freedom of conscience and free profession, practice and propagation of religion;
- freedom to manage religious affairs;
- freedom as to payment of taxes for promotion of any particular religion;
- freedom as to attendance at religious instruction or religious worship in educational institutions wholly maintained by the State.

Cultural and Educational Rights
- for protection of interests of minorities to conserve their language, script and culture;
- for minorities to establish and administer educational institutions of their choice.

Right to Constitutional Remedies
- by issuance of directions or orders or writs by the Supreme Court and High Courts for enforcement of these Fundamental Rights.

A Note for the Teachers

Mathematics has an important role in our life, it not only helps in day-to-day situations but also develops logical reasoning, abstract thinking and imagination. It enriches life and provides new dimensions to thinking. The struggle to learn abstract principles develops the power to formulate and understand arguments and the capacity to see interrelations among concepts. The enriched understanding helps us deal with abstract ideas in other subjects as well. It also helps us understand and make better patterns, maps, appreciate area and volume and see similarities between shapes and sizes. The scope of Mathematics includes many aspects of our life and our environment. This relationship needs to be brought out at all possible places.

Learning Mathematics is not about remembering solutions or methods but knowing how to solve problems. We hope that you will give your students a lot of opportunities to create and formulate problems themselves. We believe it would be a good idea to ask them to formulate as many new problems as they can. This would help children in developing an understanding of the concepts and principles of Mathematics. The nature of the problems set up by them becomes varied and more complex as they become confident with the ideas they are dealing in.

The Mathematics classroom should be alive and interactive in which the children should be articulating their own understanding of concepts, evolving models and developing definitions. Language and learning Mathematics have a very close relationship and there should be a lot of opportunity for children to talk about ideas in Mathematics and bring in their experiences in conjunction with whatever is being discussed in the classroom. There should be no obvious restriction on them using their own words and language and the shift to formal language should be gradual. There should be space for children to discuss ideas amongst themselves and make presentations as a group regarding what they have understood from the textbooks and present examples from the contexts of their own experiences. They should be encouraged to read the book in groups and formulate and express what they understand from it.

Mathematics requires abstractions. It is a discipline in which the learners learn to generalise, formulate and prove statements based on logic. In learning to abstract, children would need concrete material, experience and known context as scaffolds to help them. Please provide them with those but also ensure that they do not get over dependent on them. We may point out that the book tries to emphasise the difference between verification and proof. These two ideas are often confused and we would hope that you would take care to avoid mixing up verification with proof.

There are many situations provided in the book where children will be verifying principles or patterns and would also be trying to find out exceptions to these. So, while on the one hand children would be expected to observe patterns and make generalisations, they would also be required to identify and find exceptions to the generalisations, extend patterns to new situations and check their validity. This is an essential part of the ideas of Mathematics learning and therefore, if you can find other places where such exercises can be created for students, it would be useful. They must have many opportunities to solve problems themselves and reflect on the solutions obtained. It is hoped that you would give children the opportunity to provide logical arguments for different ideas and expect them to follow logical arguments and find loopholes in the arguments presented. This is necessary for them to develop the ability to understand what it means to prove something and also become confident about the underlying concepts.

There is expectation that in your class, Mathematics will emerge as a subject of exploration and creation rather than an exercise of finding old answers to old and complicated problems. The Mathematics classroom should not expect a blind application of ununderstood algorithm and should encourage children to find many different ways to solve problems. They need to appreciate that there are many alternative algorithms and many strategies that can be adopted to find solutions to problems. If you can include some problems that have the scope for many different correct solutions, it would help them appreciate the meaning of Mathematics better.

We have tried to link chapters with each other and to use the concepts learnt in the initial chapters to the ideas in the subsequent chapters. We hope that you will use this as an opportunity to revise these concepts in a spiraling way so that children are helped to appreciate the entire conceptual structure of Mathematics. Please give more time to ideas of negative number, fractions, variables and other ideas that are new for children. Many of these are the basis for further learning of Mathematics.

We hope that the book will help ensure that children learn to enjoy Mathematics and explore formulating patterns and problems that they will enjoy doing themselves. They should learn to be confident, not feel afraid of Mathematics and learn to help each other through discussions. We also hope that you would find time to listen carefully and identify the ideas that need to be emphasised with children and the places where the children can be given space to articulate their ideas and verbalise their thoughts. We look forward to your comments and suggestions regarding the book and hope that you will send us interesting exercises that you develop in the course of teaching so that they can be included in the next edition.

Contents

ALL MEN ARE EQUAL

"I believe implicitly that all men are born equal. All whether born in India or in England or America or in any circumstances whatsoever have the same soul as any other. And it is because I believe in this inherent equality of all men that I fight the doctrine of superiority which many arrogate to themselves."

"I have fought this doctrine of superiority in South Africa inch by inch, and it is because of that inherent belief that I delight in calling myself a scavenger, a spinner, a weaver, a farmer and a labourer."

"I consider that it is unmanly for any person to claim superiority over a fellow being. He who claims superiority, at once forfeits the claim to be called a man."

M. K. Gandhi

Such teachers still exist in India. (It should not be necessary to sound the warning that I am not speaking here of spiritual teachers who have the power to lead the aspirants to liberation.) Such teachers have no use for flattery. Respect for them must be natural and so is the love of the teacher for his pupil. That being so, the teacher is ever ready to give, and the pupil equally ready to receive. Ordinary things we may and do learn from anyone. For example, I may learn a great deal from a carpenter with whom I have nothing in common and who may even have many faults. I just buy from him the requisite knowledge even as I buy from a shopkeeper my needs. Of course, here too, a certain kind of faith is necessary. I must have faith in the knowledge of carpentry of the carpenter from whom I want to learn it. If I lack that faith, then it is clear I cannot learn anything from him. But devotion to a teacher is a different matter. Where education aims at the building of character, the old teacher-disciple relation is absolutely necessary. In the absence of a feeling of devotion to the teacher, the building of character must become difficult of achievement.

The Problem of Education : p. 155.

Knowing our Numbers

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Chapter 1

1.1 Introduction

Counting things is easy for us now. We can count objects in large numbers, for example, the number of students in the school, and represent them through numerals. We can also communicate large numbers using suitable number names.

It is not as if we always knew how to convey large quantities in conversation or through symbols. Many thousands years ago, people knew only small numbers. Gradually, they learnt how to handle larger numbers. They also learnt how to express large numbers in symbols. All this came through collective efforts of human beings. Their path was not easy, they struggled all along the way. In fact, the development of whole of Mathematics can be understood this way. As human beings progressed, there was greater need for development of Mathematics and as a result Mathematics grew further and faster.

We use numbers and know many things about them. Numbers help us count concrete objects. They help us to say which collection of objects is bigger and arrange them in order e.g., first, second, etc. Numbers are used in many different contexts and in many ways. Think about various situations where we use numbers. List five distinct situations in which numbers are used.

We enjoyed working with numbers in our previous classes. We have added, subtracted, multiplied and divided them. We also looked for patterns in number sequences and done many other interesting things with numbers. In this chapter, we shall move forward on such interesting things with a bit of review and revision as well.

1.2 Comparing Numbers

As we have done quite a lot of this earlier, let us see if we remember which is the greatest among these :

(i) 92, 392, 4456, 89742 — I am the greatest!

(ii) 1902, 1920, 9201, 9021, 9210 — I am the greatest!

So, we know the answers.
Discuss with your friends, how you find the number that is the greatest.

Try These

Can you instantly find the greatest and the smallest numbers in each row?

1. 382, 4972, 18, 59785, 750. Ans. 59785 is the greatest and 18 is the smallest.
2. 1473, 89423, 100, 5000, 310. Ans. ____________________
3. 1834, 75284, 111, 2333, 450 . Ans. ____________________
4. 2853, 7691, 9999, 12002, 124. Ans. ____________________

Was that easy? Why was it easy?

We just looked at the number of digits and found the answer. The greatest number has the most thousands and the smallest is only in hundreds or in tens.

Make five more problems of this kind and give to your friends to solve.

Now, how do we compare 4875 and 3542?

This is also not very difficult.These two numbers have the same number of digits. They are both in thousands. But the digit at the thousands place in 4875 is greater than that in 3542. Therefore, 4875 is greater than 3542.

Try These

Find the greatest and the smallest numbers.

(a) 4536, 4892, 4370, 4452.
(b) 15623, 15073, 15189, 15800.
(c) 25286, 25245, 25270, 25210.
(d) 6895, 23787, 24569, 24659.

Next tell which is greater, 4875 or 4542? Here too the numbers have the same number of digits. Further, the digits at the thousands place are same in both. What do we do then? We move to the next digit, that is to the digit at the hundreds place. The digit at the hundreds place is greater in 4875 than in 4542. Therefore, 4875 is greater than 4542.

If the digits at hundreds place are also same in the two numbers, then what do we do?

Compare 4875 and 4889 ; Also compare 4875 and 4879.

1.2.1 How many numbers can you make?

Suppose, we have four digits 7, 8, 3, 5. Using these digits we want to make different 4-digit numbers in such a way that no digit is repeated in them. Thus, 7835 is allowed, but 7735 is not. Make as many 4-digit numbers as you can.

Which is the greatest number you can get? Which is the smallest number?

The greatest number is 8753 and the smallest is 3578.

Think about the arrangement of the digits in both. Can you say how the largest number is formed? Write down your procedure.

Try These

1. Use the given digits without repetition and make the greatest and smallest 4-digit numbers.

   (a) 2, 8, 7, 4 (b) 9, 7, 4, 1 (c) 4, 7, 5, 0

   (d) 1, 7, 6, 2 (e) 5, 4, 0, 3

   (**Hint :** 0754 is a 3-digit number.)

2. Now make the greatest and the smallest 4-digit numbers by using any one digit twice.

   (a) 3, 8, 7 (b) 9, 0, 5 (c) 0, 4, 9 (d) 8, 5, 1

   (**Hint :** Think in each case which digit will you use twice.)

3. Make the greatest and the smallest 4-digit numbers using any four different digits with conditions as given.

   (a) Digit 7 is always at ones place

   Greatest

9	8	6	7

   Smallest

1	0	2	7

   (Note, the number cannot begin with the digit 0. Why?)

   (b) Digit 4 is always at tens place

   Greatest

		4	

   Smallest

		4	

   (c) Digit 9 is always at hundreds place

   Greatest

	9		

   Smallest

	9		

   (d) Digit 1 is always at thousands place

   Greatest

1			

   Smallest

1			

4. Take two digits, say 2 and 3. Make 4-digit numbers using both the digits equal number of times.

   Which is the greatest number?

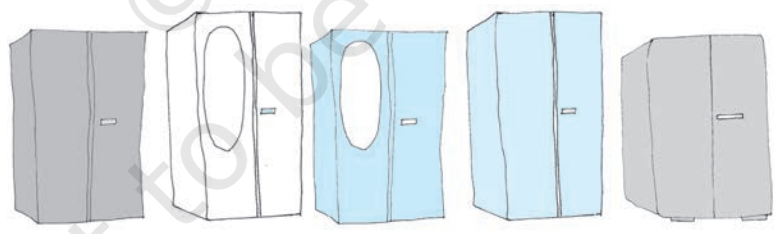
   Which is the smallest number?

   How many different numbers can you make in all?

Stand in proper order

1. Who is the tallest?
2. Who is the shortest?
   (a) Can you arrange them in the increasing order of their heights?
   (b) Can you arrange them in the decreasing order of their heights?

Ramhari (160 cm) | Dolly (154 cm) | Mohan (158 cm) | Shashi (159 cm)



₹ 2635 | ₹ 1897 | ₹ 2854 | ₹ 1788 | ₹ 3975

Which to buy?

Sohan and Rita went to buy an almirah. There were many almirahs available with their price tags.

(a) Can you arrange their prices in increasing order?

(b) Can you arrange their prices in decreasing order?

Try These

Think of five more situations where you compare three or more quantities.

Ascending order Ascending order means arrangement from the smallest to the greatest.

Descending order Descending order means arrangement from the greatest to the smallest.

Try These

1. Arrange the following numbers in ascending order :
   (a) 847, 9754, 8320, 571 (b) 9801, 25751, 36501, 38802
2. Arrange the following numbers in descending order :
   (a) 5000, 7500, 85400, 7861 (b) 1971, 45321, 88715, 92547

   Make ten such examples of ascending/descending order and solve them.

1.2.2 Shifting digits

Have you thought what fun it would be if the digits in a number could shift (move) from one place to the other?

Think about what would happen to 182. It could become as large as 821 and as small as 128. Try this with 391 as well.

Now think about this. Take any 3-digit number and exchange the digit at the hundreds place with the digit at the ones place.

(a) Is the new number greater than the former one?
(b) Is the new number smaller than the former number?

Write the numbers formed in both ascending and descending order.

Before 7 9 5

Exchanging the 1st and the 3rd tiles.

After 5 9 7

If you exchange the 1st and the 3rd tiles (i.e. digits), in which case does the number become greater? In which case does it become smaller?

Try this with a 4-digit number.

1.2.3 Introducing 10,000

We know that beyond 99 there is no 2-digit number. 99 is the greatest 2-digit number. Similarly, the greatest 3-digit number is 999 and the greatest 4-digit number is 9999. What shall we get if we add 1 to 9999?

Look at the pattern :
$9 + 1 = 10 = 10 \times 1$
$99 + 1 = 100 = 10 \times 10$
$999 + 1 = 1000 = 10 \times 100$

We observe that

Greatest single digit number + 1 = smallest 2-digit number

Greatest 2-digit number + 1 = smallest 3-digit number

Greatest 3-digit number + 1 = smallest 4-digit number

We should then expect that on adding 1 to the greatest 4-digit number, we would get the smallest 5-digit number, that is 9999 + 1 = 10000.

The new number which comes next to 9999 is 10000. It is called ten thousand. Further, 10000 = 10 × 1000.

1.2.4 Revisiting place value

You have done this quite earlier, and you will certainly remember the expansion of a 2-digit number like 78 as

$78 = 70 + 8 = 7 \times 10 + 8$

Similarly, you will remember the expansion of a 3-digit number like 278 as

$278 = 200 + 70 + 8 = 2 \times 100 + 7 \times 10 + 8$

We say, here, 8 is at ones place, 7 is at tens place and 2 at hundreds place.

Later on we extended this idea to 4-digit numbers.

For example, the expansion of 5278 is

$5278 = 5000 + 200 + 70 + 8$
$= 5 \times 1000 + 2 \times 100 + 7 \times 10 + 8$

Here, 8 is at ones place, 7 is at tens place, 2 is at hundreds place and 5 is at thousands place.

With the number 10000 known to us, we may extend the idea further. We may write 5-digit numbers like

$45278 = 4 \times 10000 + 5 \times 1000 + 2 \times 100 + 7 \times 10 + 8$

We say that here 8 is at ones place, 7 at tens place, 2 at hundreds place, 5 at thousands place and 4 at ten thousands place. The number is read as forty five thousand, two hundred seventy eight. Can you now write the smallest and the greatest 5-digit numbers?

Try These

Read and expand the numbers wherever there are blanks.

Number	Number Name	Expansion
20000	twenty thousand	2×10000
26000	twenty six thousand	$2 \times 10000 + 6 \times 1000$
38400	thirty eight thousand four hundred	$3 \times 10000 + 8 \times 1000 + 4 \times 100$
65740	sixty five thousand seven hundred forty	$6 \times 10000 + 5 \times 1000 + 7 \times 100 + 4 \times 10$

89324	eighty nine thousand three hundred twenty four	$8 \times 10000 + 9 \times 1000 + 3 \times 100 + 2 \times 10 + 4 \times 1$
50000	____________	____________
41000	____________	____________
47300	____________	____________
57630	____________	____________
29485	____________	____________
29085	____________	____________
20085	____________	____________
20005	____________	____________

Write five more 5-digit numbers, read them and expand them.

1.2.5 Introducing 1,00,000

Which is the greatest 5-digit number?

Adding 1 to the greatest 5-digit number, should give the smallest 6-digit number : 99,999 + 1 = 1,00,000

This number is named one lakh. One lakh comes next to 99,999.

$10 \times 10{,}000 = 1{,}00{,}000$

We may now write 6-digit numbers in the expanded form as

$2{,}46{,}853 = 2 \times 1{,}00{,}000 + 4 \times 10{,}000 + 6 \times 1{,}000 + 8 \times 100 + 5 \times 10 + 3 \times 1$

This number has 3 at ones place, 5 at tens place, 8 at hundreds place, 6 at thousands place, 4 at ten thousands place and 2 at lakh place. Its number name is two lakh forty six thousand eight hundred fifty three.

Try These

Read and expand the numbers wherever there are blanks.

Number	Number Name	Expansion
3,00,000	three lakh	$3 \times 1{,}00{,}000$
3,50,000	three lakh fifty thousand	$3 \times 1{,}00{,}000 + 5 \times 10{,}000$
3,53,500	three lakh fifty three thousand five hundred	$3 \times 1{,}00{,}000 + 5 \times 10{,}000 + 3 \times 1000 + 5 \times 100$
4,57,928	____________	____________
4,07,928	____________	____________
4,00,829	____________	____________
4,00,029	____________	____________

1.2.6 Larger numbers

If we add one more to the greatest 6-digit number we get the smallest 7-digit number. It is called **ten lakh.**

Write down the greatest 6-digit number and the smallest 7-digit number. Write the greatest 7-digit number and the smallest 8-digit number. The smallest 8-digit number is called **one crore.**

Complete the pattern :

9 + 1	=	10
99 + 1	=	100
999 + 1	=	________
9,999 + 1	=	________
99,999 + 1	=	________
9,99,999 + 1	=	________
99,99,999 + 1	=	1,00,00,000

Remember

1 hundred	= 10 tens
1 thousand	= 10 hundreds
	= 100 tens
1 lakh	= 100 thousands
	= 1000 hundreds
1 crore	= 100 lakhs
	= 10,000 thousands

Try These

1. What is 10 – 1 =?
2. What is 100 – 1 =?
3. What is 10,000 – 1 =?
4. What is 1,00,000 – 1 =?
5. What is 1,00,00,000 – 1 =?

(**Hint :** Use the said pattern.)

We come across large numbers in many different situations. For example, while the number of children in your class would be a 2-digit number, the number of children in your school would be a 3 or 4-digit number.

The number of people in the nearby town would be much larger.

Is it a 5 or 6 or 7-digit number?

Do you know the number of people in your state?

How many digits would that number have?

What would be the number of grains in a sack full of wheat? A 5-digit number, a 6-digit number or more?

Try These

1. Give five examples where the number of things counted would be more than 6-digit number.
2. Starting from the greatest 6-digit number, write the previous five numbers in descending order.
3. Starting from the smallest 8-digit number, write the next five numbers in ascending order and read them.

1.2.7 An aid in reading and writing large numbers

Try reading the following numbers :

(a) 279453 (b) 5035472

(c) 152700375 (d) 40350894

Was it difficult?

Did you find it difficult to keep track?

Sometimes it helps to use indicators to read and write large numbers.

Shagufta uses indicators which help her to read and write large numbers. Her indicators are also useful in writing the expansion of numbers. For example, she identifies the digits in ones place, tens place and hundreds place in 257 by writing them under the tables O, T and H as

H	T	O	Expansion
2	5	7	$2 \times 100 + 5 \times 10 + 7 \times 1$

Similarly, for 2902,

Th	H	T	O	Expansion
2	9	0	2	$2 \times 1000 + 9 \times 100 + 0 \times 10 + 2 \times 1$

One can extend this idea to numbers upto lakh as seen in the following table. (Let us call them placement boxes). Fill the entries in the blanks left.

Number	TLakh	Lakh	TTh	Th	H	T	O	Number Name	Expansion
7,34,543	—	7	3	4	5	4	3	Seven lakh thirty four thousand five hundred forty three	----------------
32,75,829	3	2	7	5	8	2	9	--------------------	$3 \times 10,00,000 + 2 \times 1,00,000 + 7 \times 10,000 + 5 \times 1000 + 8 \times 100 + 2 \times 10 + 9$

Similarly, we may include numbers upto crore as shown below :

Number	TCr	Cr	TLakh	Lakh	TTh	Th	H	T	O	Number Name
2,57,34,543	—	2	5	7	3	4	5	4	3	....
65,32,75,829	6	5	3	2	7	5	8	2	9	Sixty five crore thirty two lakh seventy five thousand eight hundred twenty nine

You can make other formats of tables for writing the numbers in expanded form.

Use of commas

You must have noticed that in writing large numbers in the sections above, we have often used commas. Commas help us in reading and writing large numbers. In our **Indian System of Numeration** we use ones, tens, hundreds, thousands and then lakhs and crores. Commas are used to mark thousands, lakhs and crores. The first comma comes after hundreds place (three digits from the right) and marks thousands. The second comma comes two digits later (five digits from the right). It comes after ten thousands place and marks lakh. The third comma comes after another two digits (seven digits from the right). It comes after ten lakh place and marks crore.

> While writing number names, we do not use commas.

For example, 5, 08, 01, 592

3, 32, 40, 781

7, 27, 05, 062

Try reading the numbers given above. Write five more numbers in this form and read them.

International System of Numeration

In the International System of Numeration, as it is being used we have ones, tens, hundreds, thousands and then millions. One million is a thousand thousands. Commas are used to mark thousands and millions. It comes after every three digits from the right. The first comma marks thousands and the next comma marks millions. For example, the number 50,801,592 is read in the International System as fifty million eight hundred one thousand five hundred ninety two. In the Indian System, it is five crore eight lakh one thousand five hundred ninety two.

How many lakhs make a million?

How many millions make a crore?

Take three large numbers. Express them in both Indian and International Numeration systems.

Interesting fact :

To express numbers larger than a million, a billion is used in the International System of Numeration: 1 billion = 1000 million.

Do you know?
India's population increased by about
27 million during 1921-1931;
37 million during 1931-1941;
44 million during 1941-1951;
78 million during 1951-1961!

How much was the increase in population during 1991-2001? Try to find out.

Do you know what is India's population today? Try to find this too.

Try These

1. Read these numbers. Write them using placement boxes and then write their expanded forms.

   (i) 475320 (ii) 9847215 (iii) 97645310 (iv) 30458094

   (a) Which is the smallest number?

   (b) Which is the greatest number?

   (c) Arrange these numbers in ascending and descending orders.

2. Read these numbers.

   (i) 527864 (ii) 95432 (iii) 18950049 (iv) 70002509

   (a) Write these numbers using placement boxes and then using commas in Indian as well as International System of Numeration..

   (b) Arrange these in ascending and descending order.

3. Take three more groups of large numbers and do the exercise given above.

Can you help me write the numeral?

To write the numeral for a number you can follow the boxes again.

(a) Forty two lakh seventy thousand eight.

(b) Two crore ninety lakh fifty five thousand eight hundred.

(c) Seven crore sixty thousand fifty five.

Try These

1. You have the following digits 4, 5, 6, 0, 7 and 8. Using them, make five numbers each with 6 digits.

   (a) Put commas for easy reading.

   (b) Arrange them in ascending and descending order.

2. Take the digits 4, 5, 6, 7, 8 and 9. Make any three numbers each with 8 digits. Put commas for easy reading.

3. From the digits 3, 0 and 4, make five numbers each with 6 digits. Use commas.

EXERCISE 1.1

1. Fill in the blanks:
   (a) 1 lakh = ________ ten thousand.
   (b) 1 million = ________ hundred thousand.
   (c) 1 crore = ________ ten lakh.
   (d) 1 crore = ________ million.
   (e) 1 million = ________ lakh.

2. Place commas correctly and write the numerals:
   (a) Seventy three lakh seventy five thousand three hundred seven.
   (b) Nine crore five lakh forty one.
   (c) Seven crore fifty two lakh twenty one thousand three hundred two.
   (d) Fifty eight million four hundred twenty three thousand two hundred two.
   (e) Twenty three lakh thirty thousand ten.

3. Insert commas suitably and write the names according to Indian System of Numeration :
   (a) 87595762 (b) 8546283 (c) 99900046 (d) 98432701

4. Insert commas suitably and write the names according to International System of Numeration :
   (a) 78921092 (b) 7452283 (c) 99985102 (d) 48049831

1.3 Large Numbers in Practice

In earlier classes, we have learnt that we use centimetre (cm) as a unit of length. For measuring the length of a pencil, the width of a book or notebooks etc., we use centimetres. Our ruler has marks on each centimetre.

For measuring the thickness of a pencil, however, we find centimetre too big.

We use millimetre (mm) to show the thickness of a pencil.

(a) 10 millimetres = 1 centimetre

To measure the length of the classroom or the school building, we shall find centimetre too small. We use metre for the purpose.

(b) 1 metre = 100 centimetres

= 1000 millimetres

Even metre is too small, when we have to state distances between cities, say, Delhi and Mumbai, or Chennai and Kolkata. For this we need kilometres (km).

Try These

1. How many centimetres make a kilometre?
2. Name five large cities in India. Find their population. Also, find the distance in kilometres between each pair of these cities.

(c) 1 kilometre = 1000 metres

How many millimetres make 1 kilometre?

Since 1 m = 1000 mm

1 km = 1000 m = 1000 × 1000 mm = 10,00,000 mm

We go to the market to buy rice or wheat; we buy it in kilograms (kg). But items like ginger or chillies which we do not need in large quantities, we buy in grams (g). We know 1 kilogram = 1000 grams.

Have you noticed the weight of the medicine tablets given to the sick? It is very small. It is in milligrams (mg).

1 gram = 1000 milligrams.

What is the capacity of a bucket for holding water? It is usually 20 litres (ℓ). Capacity is given in litres. But sometimes we need a smaller unit, the millilitres. A bottle of hair oil, a cleaning liquid or a soft drink have labels which give the quantity of liquid inside in millilitres (ml).

1 litre = 1000 millilitres.

Note that in all these units we have some words common like kilo, milli and centi. You should remember that among these **kilo** is the greatest and **milli** is the smallest; kilo shows 1000 times greater, milli shows 1000 times smaller, i.e. 1 kilogram = 1000 grams, 1 gram = 1000 milligrams.

Try These

1. How many milligrams make one kilogram?
2. A box contains 2,00,000 medicine tablets each weighing 20 mg. What is the total weight of all the tablets in the box in grams and in kilograms?

Similarly, centi shows 100 times smaller, i.e. 1 metre = 100 centimetres.

Try These

1. A bus started its journey and reached different places with a speed of 60 km/hour. The journey is shown on page 14.

   (i) Find the total distance covered by the bus from A to D.

   (ii) Find the total distance covered by the bus from D to G.

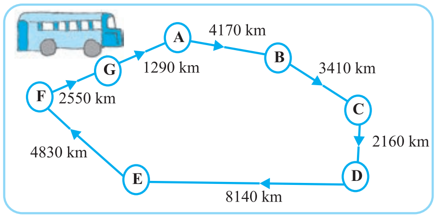
   (iii) Find the total distance covered by the bus, if it starts from A and returns back to A.

   (iv) Can you find the difference of distances from C to D and D to E?

(v) Find out the time taken by the bus to reach

(a) A to B (b) C to D

(c) E to G (d) Total journey



2. **Raman's shop**

Things	Price
Apples	₹ 40 per kg
Oranges	₹ 30 per kg
Combs	₹ 3 for one
Tooth brushes	₹ 10 for one
Pencils	₹ 1 for one
Note books	₹ 6 for one
Soap cakes	₹ 8 for one

The sales during the last year

Apples	2457 kg
Oranges	3004 kg
Combs	22760
Tooth brushes	25367
Pencils	38530
Note books	40002
Soap cakes	20005

(a) Can you find the total weight of apples and oranges Raman sold last year?

Weight of apples = __________ kg

Weight of oranges = _________ kg

Therefore, total weight = _____ kg + _____ kg = _____ kg

Answer – The total weight of oranges and apples = _________ kg.

(b) Can you find the total money Raman got by selling apples?

(c) Can you find the total money Raman got by selling apples and oranges together?

(d) Make a table showing how much money Raman received from selling each item. Arrange the entries of amount of money received in descending order. Find the item which brought him the highest amount. How much is this amount?

We have done a lot of problems that have addition, subtraction, multiplication and division. We will try solving some more here. Before starting, look at these examples and follow the methods used.

Example 1 : Population of Sundarnagar was 2,35,471 in the year 1991. In the year 2001 it was found to be increased by 72,958. What was the population of the city in 2001?

Solution : Population of the city in 2001

= Population of the city in 1991 + Increase in population

= 2,35,471 + 72,958

Now,

$$\begin{array}{r} 235471 \\ +\ 72958 \\ \hline 308429 \\ \hline \end{array}$$

Salma added them by writing 235471 as 200000 + 35000 + 471 and 72958 as 72000 + 958. She got the addition as 200000 + 107000 + 1429 = 308429. Mary added it as 200000 + 35000 + 400 + 71 + 72000 + 900 + 58 = 308429

Answer : Population of the city in 2001 was 3,08,429.

All three methods are correct.

Example 2 : In one state, the number of bicycles sold in the year 2002-2003 was 7,43,000. In the year 2003-2004, the number of bicycles sold was 8,00,100. In which year were more bicycles sold? and how many more?

Solution : Clearly, 8,00,100 is more than 7,43,000. So, in that state, more bicycles were sold in the year 2003-2004 than in 2002-2003.

Now,

$$\begin{array}{r} 800100 \\ -\ 743000 \\ \hline 057100 \\ \hline \end{array}$$

Check the answer by adding

$$\begin{array}{r} 743000 \\ +\ 57100 \\ \hline 800100 \\ \hline \end{array}$$

(the answer is right)

Can you think of alternative ways of solving this problem?

Answer : 57,100 more bicycles were sold in the year 2003-2004.

Example 3 : The town newspaper is published every day. One copy has 12 pages. Everyday 11,980 copies are printed. How many total pages are printed everyday?

Solution : Each copy has 12 pages. Hence, 11,980 copies will have $12 \times 11{,}980$ pages. What would this number be? More than 1,00,000 or lesser. Try to estimate.

Now,

$$\begin{array}{r} 11980 \\ \times\ 12 \\ \hline 23960 \\ +\ 119800 \\ \hline 143760 \\ \hline \end{array}$$

Answer: Everyday 1,43,760 pages are printed.

Example 4 : The number of sheets of paper available for making notebooks is 75,000. Each sheet makes 8 pages of a notebook. Each notebook contains 200 pages. How many notebooks can be made from the paper available?

Solution : Each sheet makes 8 pages.

Hence, 75,000 sheets make $8 \times 75{,}000$ pages,

Now,

$$\begin{array}{r} 75000 \\ \times\ 8 \\ \hline 600000 \\ \hline \end{array}$$

Thus, 6,00,000 pages are available for making notebooks.
Now, 200 pages make 1 notebook.
Hence, 6,00,000 pages make $6{,}00{,}000 \div 200$ notebooks.

Now,

$$\begin{array}{r|l} & 3000 \\ \hline 200 & 600000 \\ & -\ 600 \\ \hline & 0000 \end{array}$$

The answer is 3,000 notebooks.

EXERCISE 1.2

1. A book exhibition was held for four days in a school. The number of tickets sold at the counter on the first, second, third and final day was respectively 1094, 1812, 2050 and 2751. Find the total number of tickets sold on all the four days.
2. Shekhar is a famous cricket player. He has so far scored 6980 runs in test matches. He wishes to complete 10,000 runs. How many more runs does he need?
3. In an election, the successful candidate registered 5,77,500 votes and his nearest rival secured 3,48,700 votes. By what margin did the successful candidate win the election?

4. Kirti bookstore sold books worth ₹ 2,85,891 in the first week of June and books worth ₹ 4,00,768 in the second week of the month. How much was the sale for the two weeks together? In which week was the sale greater and by how much?
5. Find the difference between the greatest and the least 5-digit number that can be written using the digits 6, 2, 7, 4, 3 each only once.
6. A machine, on an average, manufactures 2,825 screws a day. How many screws did it produce in the month of January 2006?
7. A merchant had ₹ 78,592 with her. She placed an order for purchasing 40 radio sets at ₹ 1200 each. How much money will remain with her after the purchase?
8. A student multiplied 7236 by 65 instead of multiplying by 56. By how much was his answer greater than the correct answer? (**Hint:** Do you need to do both the multiplications?)
9. To stitch a shirt, 2 m 15 cm cloth is needed. Out of 40 m cloth, how many shirts can be stitched and how much cloth will remain?

   (Hint: convert data in cm.)
10. Medicine is packed in boxes, each weighing 4 kg 500g. How many such boxes can be loaded in a van which cannot carry beyond 800 kg?
11. The distance between the school and a student's house is 1 km 875 m. Everyday she walks both ways. Find the total distance covered by her in six days.
12. A vessel has 4 litres and 500 ml of curd. In how many glasses, each of 25 ml capacity, can it be filled?

What have we discussed?

1. Given two numbers, one with more digits is the greater number. If the number of digits in two given numbers is the same, that number is larger, which has a greater leftmost digit. If this digit also happens to be the same, we look at the next digit and so on.
2. In forming numbers from given digits, we should be careful to see if the conditions under which the numbers are to be formed are satisfied. Thus, to form the greatest four digit number from 7, 8, 3, 5 without repeating a single digit, we need to use all four digits, the greatest number can have only 8 as the leftmost digit.
3. The smallest four digit number is 1000 (one thousand). It follows the largest three digit number 999. Similarly, the smallest five digit number is 10,000. It is ten thousand and follows the largest four digit number 9999.

   Further, the smallest six digit number is 100,000. It is one lakh and follows the largest five digit number 99,999. This carries on for higher digit numbers in a similar manner.
4. Use of commas helps in reading and writing large numbers. In the Indian system of numeration we have commas after 3 digits starting from the right and thereafter every 2 digits. The commas after 3, 5 and 7 digits separate thousand, lakh and crore

respectively. In the International system of numeration commas are placed after every 3 digits starting from the right. The commas after 3 and 6 digits separate thousand and million respectively.

5. Large numbers are needed in many places in daily life. For example, for giving number of students in a school, number of people in a village or town, money paid or received in large transactions (paying and selling), in measuring large distances say betwen various cities in a country or in the world and so on.

6. Remember kilo shows 1000 times larger, Centi shows 100 times smaller and milli shows 1000 times smaller, thus, 1 kilometre = 1000 metres, 1 metre = 100 centimetres or 1000 millimetres etc.

Whole Numbers

Chapter 2

0650CH02

2.1 Introduction

As we know, we use 1, 2, 3, 4,... when we begin to count. They come naturally when we start counting. Hence, mathematicians call the counting numbers as Natural numbers.

Predecessor and successor

Given any natural number, you can add 1 to that number and get the next number i.e. you get its successor.

The successor of 16 is 16 + 1 = 17, that of 19 is 19 +1 = 20 and so on.

The number 16 comes before 17, we say that the predecessor of 17 is 17–1=16, the predecessor of 20 is 20 – 1 = 19, and so on.

The number 3 has a predecessor and a successor. What about 2? The successor is 3 and the predecessor is 1. Does 1 have both a successor and a predecessor?

Try These

1. Write the predecessor and successor of 19; 1997; 12000; 49; 100000.
2. Is there any natural number that has no predecessor?
3. Is there any natural number which has no successor? Is there a last natural number?

We can count the number of children in our school; we can also count the number of people in a city; we can count the number of people in India. The number of people in the whole world can also be counted. We may not be able to count the number of stars in the sky or the number of hair on our heads but if we are able, there would be a number for them also. We can then add one more to such a number and

get a larger number. In that case we can even write the number of hair on two heads taken together.

It is now perhaps obvious that there is no largest number. Apart from these questions shared above, there are many others that can come to our mind when we work with natural numbers. You can think of a few such questions and discuss them with your friends. You may not clearly know the answers to many of them !

2.2 Whole Numbers

We have seen that the number 1 has no predecessor in natural numbers. To the collection of natural numbers we add zero as the predecessor for 1.

> The natural numbers along with zero form the collection of whole numbers.

Try These

1. Are all natural numbers also whole numbers?
2. Are all whole numbers also natural numbers?
3. Which is the greatest whole number?

In your previous classes you have learnt to perform all the basic operations like addition, subtraction, multiplication and division on numbers. You also know how to apply them to problems. Let us try them on a number line. Before we proceed, let us find out what a number line is!

2.3 The Number Line

Draw a line. Mark a point on it. Label it 0. Mark a second point to the right of 0. Label it 1.

The distance between these points labelled as 0 and 1 is called unit distance. On this line, mark a point to the right of 1 and at unit distance from 1 and label it 2. In this way go on labelling points at unit distances as 3, 4, 5,... on the line. You can go to any whole number on the right in this manner.

This is a number line for the whole numbers.

What is the distance between the points 2 and 4? Certainly, it is 2 units. Can you tell the distance between the points 2 and 6, between 2 and 7?

On the number line you will see that the number 7 is on the right of 4. This number 7 is greater than 4, i.e. 7 > 4. The number 8 lies on the right of 6 and 8 > 6. These observations help us to say that, out of any two whole numbers, the number on the right of the other number is the greater number. We can also say that whole number on left is the smaller number.

For example, 4 < 9; 4 is on the left of 9. Similarly, 12 > 5; 12 is to the right of 5.

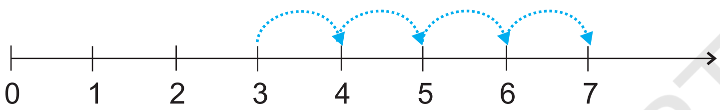
What can you say about 10 and 20?

Mark 30, 12, 18 on the number line. Which number is at the farthest left? Can you say from 1005 and 9756, which number would be on the right relative to the other number.

Place the successor of 12 and the predecessor of 7 on the number line.

Addition on the number line

Addition of whole numbers can be shown on the number line. Let us see the addition of 3 and 4.



Start from 3. Since we add 4 to this number so we make 4 jumps to the right; from 3 to 4, 4 to 5, 5 to 6 and 6 to 7 as shown above. The tip of the last arrow in the fourth jump is at 7.

The sum of 3 and 4 is 7, i.e. 3 + 4 = 7.

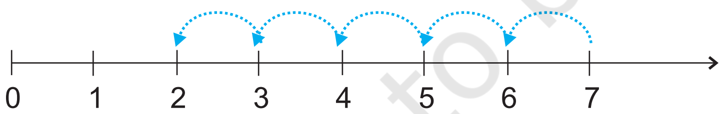
Try These

Find 4 + 5; 2 + 6; 3 + 5 and 1+6 using the number line.

Subtraction on the number line

The subtraction of two whole numbers can also be shown on the number line.

Let us find 7 – 5.



Start from 7. Since 5 is being subtracted, so move towards left with 1 jump of 1 unit. Make 5 such jumps. We reach the point 2. We get 7 – 5 = 2.

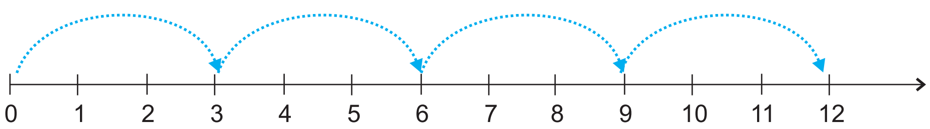
Try These

Find 8 – 3; 6 – 2; 9 – 6 using the number line.

Multiplication on the number line

We now see the multiplication of whole numbers on the number line.

Let us find 4 × 3.



Start from 0, move 3 units at a time to the right, make such 4 moves. Where do you reach? You will reach 12. So, we say, $3 \times 4 = 12$.

Try These

Find 2×6; 3×3; 4×2 using the number line.

EXERCISE 2.1

1. Write the next three natural numbers after 10999.
2. Write the three whole numbers occurring just before 10001.
3. Which is the smallest whole number?
4. How many whole numbers are there between 32 and 53?
5. Write the successor of :

   (a) 2440701 (b) 100199 (c) 1099999 (d) 2345670
6. Write the predecessor of :

   (a) 94 (b) 10000 (c) 208090 (d) 7654321
7. In each of the following pairs of numbers, state which whole number is on the left of the other number on the number line. Also write them with the appropriate sign (>, <) between them.

   (a) 530, 503 (b) 370, 307 (c) 98765, 56789 (d) 9830415, 10023001
8. Which of the following statements are true (T) and which are false (F) ?

   (a) Zero is the smallest natural number.
   (b) 400 is the predecessor of 399.
   (c) Zero is the smallest whole number.
   (d) 600 is the successor of 599.
   (e) All natural numbers are whole numbers.
   (f) All whole numbers are natural numbers.
   (g) The predecessor of a two digit number is never a single digit number.
   (h) 1 is the smallest whole number.
   (i) The natural number 1 has no predecessor.
   (j) The whole number 1 has no predecessor.
   (k) The whole number 13 lies between 11 and 12.
   (l) The whole number 0 has no predecessor.
   (m) The successor of a two digit number is always a two digit number.

What have we discussed?

1. The numbers 1, 2, 3,... which we use for counting are known as natural numbers.
2. If you add 1 to a natural number, we get its successor. If you subtract 1 from a natural number, you get its predecessor.

3. Every natural number has a successor. Every natural number except 1 has a predecessor.
4. If we add the number zero to the collection of natural numbers, we get the collection of whole numbers. Thus, the numbers 0, 1, 2, 3,... form the collection of whole numbers.
5. Every whole number has a successor. Every whole number except zero has a predecessor.
6. All natural numbers are whole numbers, but all whole numbers are not natural numbers.
7. We take a line, mark a point on it and label it 0. We then mark out points to the right of 0, at equal intervals. Label them as 1, 2, 3,.... Thus, we have a number line with the whole numbers represented on it. We can easily perform the number operations of addition, subtraction and multiplication on the number line.
8. Addition corresponds to moving to the right on the number line, whereas subtraction corresponds to moving to the left. Multiplication corresponds to making jumps of equal distance starting from zero.

Playing with Numbers

0650CH03

Chapter 3

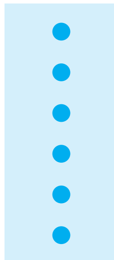
3.1 Introduction

Ramesh has 6 marbles with him. He wants to arrange them in rows in such a way that each row has the same number of marbles. He arranges them in the following ways and matches the total number of marbles.

(i) 1 marble in each row

Number of rows $= 6$

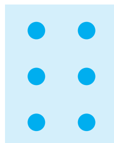
Total number of marbles $= 1 \times 6 = 6$



(ii) 2 marbles in each row

Number of rows $= 3$

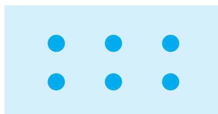
Total number of marbles $= 2 \times 3 = 6$



(iii) 3 marbles in each row

Number of rows $= 2$

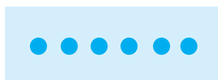
Total number of marbles $= 3 \times 2 = 6$



(iv) He could not think of any arrangement in which each row had 4 marbles or 5 marbles. So, the only possible arrangement left was with all the 6 marbles in a row.

Number of rows $= 1$

Total number of marbles $= 6 \times 1 = 6$



From these calculations Ramesh observes that 6 can be written as a product of two numbers in different ways as

$6 = 1 \times 6$; $6 = 2 \times 3$; $6 = 3 \times 2$; $6 = 6 \times 1$

From 6 = 2 × 3 it can be said that 2 and 3 exactly divide 6. So, 2 and 3 are exact divisors of 6. From the other product 6 = 1 × 6, the exact divisors of 6 are found to be 1 and 6.

Thus, 1, 2, 3 and 6 are exact divisors of 6. They are called the **factors** of 6. Try arranging 18 marbles in rows and find the factors of 18.

3.2 Factors and Multiples

Mary wants to find those numbers which exactly divide 4. She divides 4 by numbers less than 4 this way.

1)	2)	3)	4)
4 (4 – 4 0	4 (2 – 4 0	4 (1 – 3 1	4 (1 – 4 0
Quotient is 4	Quotient is 2	Quotient is 1	Quotient is 1
Remainder is 0	Remainder is 0	Remainder is 1	Remainder is 0
4 = 1 × 4	4 = 2 × 2		4 = 4 × 1

She finds that the number 4 can be written as: 4 = 1 × 4; 4 = 2 × 2; 4 = 4 × 1 and knows that the numbers 1, 2 and 4 are exact divisors of 4.

These numbers are called factors of 4.

A factor of a number is an exact divisor of that number.

Observe each of the factors of 4 is less than or equal to 4.

Game-1 : This is a game to be played by two persons say A and B. It is about spotting factors.

It requires 50 pieces of cards numbered 1 to 50.

Arrange the cards on the table like this.

1	2	3	4	5	6	7	
8	9	10	11	12	13	14	
15	16	17	18	19	20	21	
22	23	24	25	26	27	28	
29	30	31	32	33	34	35	
36	37	38	39	40	41	42	
43	44	45	46	47	48	49	50

Rationalised 2023-24

Steps

(a) Decide who plays first, A or B.

(b) Let A play first. He picks up a card from the table, and keeps it with him. Suppose the card has number 28 on it.

(c) Player B then picks up all those cards having numbers which are factors of the number on A's card (i.e. 28), and puts them in a pile near him.

(d) Player B then picks up a card from the table and keeps it with him. From the cards that are left, A picks up all those cards whose numbers are factors of the number on B's card. A puts them on the previous card that he collected.

(e) The game continues like this until all the cards are used up.

(f) A will add up the numbers on the cards that he has collected. B too will do the same with his cards. The player with greater sum will be the winner.

The game can be made more interesting by increasing the number of cards. Play this game with your friend. Can you find some way to win the game?

When we write a number 20 as $20 = 4 \times 5$, we say 4 and 5 are factors of 20. We also say that 20 is a multiple of 4 and 5.

The representation $24 = 2 \times 12$ shows that 2 and 12 are factors of 24, whereas 24 is a multiple of 2 and 12.

multiple
↑
$4 \times 5 = 20$
↓ ↓
factor factor

We can say that a number is a multiple of each of its factors

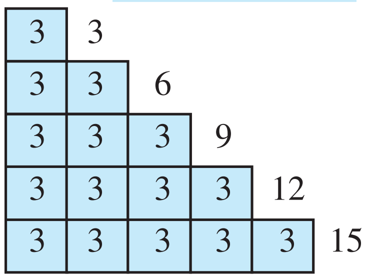
Let us now see some interesting facts about factors and multiples.

Try These

Find the possible factors of 45, 30 and 36.

(a) Collect a number of wooden/paper strips of length 3 units each.

(b) Join them end to end as shown in the following figure.



The length of the strip at the top is $3 = 1 \times 3$ units.

The length of the strip below it is $3 + 3 = 6$ units. Also, $6 = 2 \times 3$. The length of the next strip is $3 + 3 + 3 = 9$ units, and $9 = 3 \times 3$. Continuing this way we can express the other lengths as,

$12 = 4 \times 3$; $15 = 5 \times 3$

We say that the numbers 3, 6, 9, 12, 15 are multiples of 3.

The list of multiples of 3 can be continued as 18, 21, 24, ...

Each of these multiples is greater than or equal to 3.

The multiples of the number 4 are 4, 8, 12, 16, 20, 24, ...

The list is endless. Each of these numbers is greater than or equal to 4.

Let us see what we conclude about factors and multiples:

1. Is there any number which occurs as a factor of every number ? Yes. It is 1. For example 6 = 1 × 6, 18 = 1 × 18 and so on. Check it for a few more numbers.

   We say **1 is a factor of every number.**

2. Can 7 be a factor of itself ? Yes. You can write 7 as 7 = 7 × 1. What about 10? and 15?.

   You will find that every number can be expressed in this way.

   We say that **every number is a factor of itself.**

3. What are the factors of 16? They are 1, 2, 4, 8, 16. Out of these factors do you find any factor which does not divide 16? Try it for 20; 36.

   You will find that **every factor of a number is an exact divisor of that number.**

4. What are the factors of 34? They are 1, 2, 17 and 34 itself. Out of these which is the greatest factor? It is 34 itself.

   The other factors 1, 2 and 17 are less than 34. Try to check this for 64, 81 and 56.

   We say that **every factor is less than or equal to the given number.**

5. The number 76 has 5 factors. How many factors does 136 or 96 have? You will find that you are able to count the number of factors of each of these.

   Even if the numbers are as large as 10576, 25642 etc. or larger, you can still count the number of factors of such numbers, (though you may find it difficult to factorise such numbers).

   We say that **number of factors of a given number are finite.**

6. What are the multiples of 7? Obviously, 7, 14, 21, 28,... You will find that each of these multiples is greater than or equal to 7. Will it happen with each number? Check this for the multiples of 6, 9 and 10.

   We find that **every multiple of a number is greater than or equal to that number.**

7. Write the multiples of 5. They are 5, 10, 15, 20, ... Do you think this list will end anywhere? No! The list is endless. Try it with multiples of 6,7 etc.

   We find that **the number of multiples of a given number is infinite.**

8. Can 7 be a multiple of itself ? Yes, because 7 = 7×1. Will it be true for other numbers also? Try it with 3, 12 and 16.

   You will find that **every number is a multiple of itself.**

The factors of 6 are 1, 2, 3 and 6. Also, 1+2+3+6 = 12 = 2 × 6. We find that the sum of the factors of 6 is twice the number 6. All the factors of 28 are 1, 2, 4, 7, 14 and 28. Adding these we have, 1 + 2 + 4 + 7 + 14 + 28 = 56 = 2 × 28.

The sum of the factors of 28 is equal to twice the number 28.

A number for which sum of all its factors is equal to twice the number is called a perfect number. The numbers 6 and 28 are perfect numbers.

Is 10 a perfect number?

Example 1 : Write all the factors of 68.

Solution : We note that

68 = 1 × 68   68 = 2 × 34

68 = 4 × 17   68 = 17 × 4

Stop here, because 4 and 17 have occurred earlier.

Thus, all the factors of 68 are 1, 2, 4, 17, 34 and 68.

Example 2 : Find the factors of 36.

Solution : 36 = 1 × 36   36 = 2 × 18   36 = 3 × 12

36 = 4 × 9   36 = 6 × 6

Stop here, because both the factors (6) are same. Thus, the factors are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Example 3 : Write first five multiples of 6.

Solution : The required multiples are: 6×1= 6, 6×2 = 12, 6×3 = 18, 6×4 = 24, 6×5 = 30 i.e. 6, 12, 18, 24 and 30.

EXERCISE 3.1

1. Write all the factors of the following numbers :

   (a) 24   (b) 15   (c) 21

   (d) 27   (e) 12   (f) 20

   (g) 18   (h) 23   (i) 36

2. Write first five multiples of :

   (a) 5   (b) 8   (c) 9

3. Match the items in column 1 with the items in column 2.

Column 1	Column 2
(i) 35	(a) Multiple of 8
(ii) 15	(b) Multiple of 7
(iii) 16	(c) Multiple of 70
(iv) 20	(d) Factor of 30

(v) 25 (e) Factor of 50

(f) Factor of 20

4. Find all the multiples of 9 upto 100.

3.3 Prime and Composite Numbers

We are now familiar with the factors of a number. Observe the number of factors of a few numbers arranged in this table.

Numbers	Factors	Number of Factors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6

We find that (a) The number 1 has only one factor (i.e. itself).

(b) There are numbers, having exactly two factors 1 and the number itself. Such number are 2, 3, 5, 7, 11 etc. These numbers are prime numbers.

The numbers other than 1 whose only factors are 1 and the number itself are called Prime numbers.

Try to find some more prime numbers other than these.

(c) There are numbers having more than two factors like 4, 6, 8, 9, 10 and so on. These numbers are composite numbers.

> 1 is neither a prime nor a composite number.

Numbers having more than two factors are called Composite numbers.

Is 15 a composite number? Why? What about 18? 25?

Without actually checking the factors of a number, we can find prime numbers from 1 to 100 with an easier method. This method was given by a

Greek Mathematician **Eratosthenes,** in the third century B.C. Let us see the method. List all numbers from 1 to 100, as shown below.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Step 1 : Cross out 1 because it is not a prime number.

Step 2 : Encircle 2, cross out all the multiples of 2, other than 2 itself, i.e. 4, 6, 8 and so on.

Step 3 : You will find that the next uncrossed number is 3. Encircle 3 and cross out all the multiples of 3, other than 3 itself.

Step 4 : The next uncrossed number is 5. Encircle 5 and cross out all the multiples of 5 other than 5 itself.

Step 5 : Continue this process till all the numbers in the list are either encircled or crossed out.

All the encircled numbers are prime numbers. All the crossed out numbers, other than 1 are composite numbers.

This method is called the **Sieve of Eratosthenes**.

Try These

Observe that $2 \times 3 + 1 = 7$ is a prime number. Here, 1 has been added to a multiple of 2 to get a prime number. Can you find some more numbers of this type?

Example 4 : Write all the prime numbers less than 15.

Solution : By observing the Sieve Method, we can easily write the required prime numbers as 2, 3, 5, 7, 11 and 13.

even and odd numbers

Do you observe any pattern in the numbers 2, 4, 6, 8, 10, 12, 14, ...? You will find that each of them is a multiple of 2.

These are called *even numbers*. The rest of the numbers 1, 3, 5, 7, 9, 11,... are called *odd numbers*.

You can verify that a two digit number or a three digit number is even or not. How will you know whether a number like 756482 is even? By dividing it by 2. Will it not be tedious?

We say that a number with 0, 2, 4, 6, 8 at the ones place is an *even number*. So, 350, 4862, 59246 are even numbers. The numbers 457, 2359, 8231 are all odd. Let us try to find some interesting facts:

(a) Which is the smallest even number? It is 2. Which is the smallest prime number? It is again 2.

Thus, **2 is the smallest prime number which is even.**

(b) The other prime numbers are 3, 5, 7, 11, 13, ... Do you find any even number in this list? Of course not, they are all odd.
Thus, we can say that **every prime number except 2 is odd.**

EXERCISE 3.2

1. What is the sum of any two (a) Odd numbers? (b) Even numbers?
2. State whether the following statements are True or False:
   (a) The sum of three odd numbers is even.
   (b) The sum of two odd numbers and one even number is even.
   (c) The product of three odd numbers is odd.
   (d) If an even number is divided by 2, the quotient is always odd.
   (e) All prime numbers are odd.
   (f) Prime numbers do not have any factors.
   (g) Sum of two prime numbers is always even.
   (h) 2 is the only even prime number.
   (i) All even numbers are composite numbers.
   (j) The product of two even numbers is always even.
3. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers upto 100.
4. Write down separately the prime and composite numbers less than 20.
5. What is the greatest prime number between 1 and 10?
6. Express the following as the sum of two odd primes.
   (a) 44 (b) 36 (c) 24 (d) 18
7. Give three pairs of prime numbers whose difference is 2.
   [Remark : Two prime numbers whose difference is 2 are called twin primes].
8. Which of the following numbers are prime?
   (a) 23 (b) 51 (c) 37 (d) 26
9. Write seven consecutive composite numbers less than 100 so that there is no prime number between them.

10. Express each of the following numbers as the sum of three odd primes:

    (a) 21 (b) 31 (c) 53 (d) 61

11. Write five pairs of prime numbers less than 20 whose sum is divisible by 5.

    (**Hint :** 3+7 = 10)

12. Fill in the blanks :
    - (a) A number which has only two factors is called a _______.
    - (b) A number which has more than two factors is called a _______.
    - (c) 1 is neither _______ nor _______.
    - (d) The smallest prime number is _______.
    - (e) The smallest composite number is ______.
    - (f) The smallest even number is _______.

3.4 Tests for Divisibility of Numbers

Is the number 38 divisible by 2? by 4? by 5?

By actually dividing 38 by these numbers we find that it is divisible by 2 but not by 4 and by 5.

Let us see whether we can find a pattern that can tell us whether a number is divisible by 2, 3, 4, 5, 6, 8, 9, 10 or 11. Do you think such patterns can be easily seen?

Divisibility by 10 : Charu was looking at the multiples of 10. The multiples are 10, 20, 30, 40, 50, 60, ... She found something common in these numbers. Can you tell what? Each of these numbers has 0 in the ones place.

She thought of some more numbers with 0 at ones place like 100, 1000, 3200, 7010. She also found that all such numbers are divisible by 10.

She finds that **if a number has 0 in the ones place then it is divisible by 10.**

Can you find out the divisibility rule for 100?

Divisibility by 5 : Mani found some interesting pattern in the numbers 5, 10, 15, 20, 25, 30, 35, ... Can you tell the pattern? Look at the units place. All these numbers have either 0 or 5 in their ones place. We know that these numbers are divisible by 5.

Mani took up some more numbers that are divisible by 5, like 105, 215, 6205, 3500. Again these numbers have either 0 or 5 in their ones places.

He tried to divide the numbers 23, 56, 97 by 5. Will he be able to do that? Check it. He observes that **a number which has either 0 or 5 in its ones place is divisible by 5,** other numbers leave a remainder.

Is 1750125 divisible 5?

Divisibility by 2 : Charu observes a few multiples of 2 to be 10, 12, 14, 16... and also numbers like 2410, 4356, 1358, 2972, 5974. She finds some pattern in the ones place of these numbers. Can you tell that? These numbers have only the digits 0, 2, 4, 6, 8 in the ones place.

She divides these numbers by 2 and gets remainder 0.

She also finds that the numbers 2467, 4829 are not divisible by 2. These numbers do not have 0, 2, 4, 6 or 8 in their ones place.

Looking at these observations she concludes that **a number is divisible by 2 if it has any of the digits 0, 2, 4, 6 or 8 in its ones place.**

Divisibility by 3 : Are the numbers 21, 27, 36, 54, 219 divisible by 3? Yes, they are.

Are the numbers 25, 37, 260 divisible by 3? No.

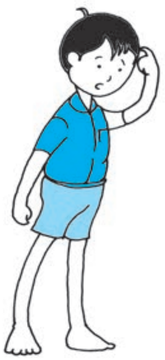
Can you see any pattern in the ones place? We cannot, because numbers with the same digit in the ones places can be divisible by 3, like 27, or may not be divisible by 3 like 17, 37. Let us now try to add the digits of 21, 36, 54 and 219. Do you observe anything special ? 2+1=3, 3+6=9, 5+4=9, 2+1+9=12. All these additions are divisible by 3.

Add the digits in 25, 37, 260. We get 2+5=7, 3+7=10, 2+6+0 = 8.

These are not divisible by 3.

We say that **if the sum of the digits is a multiple of 3, then the number is divisible by 3.**

Is 7221 divisible by 3?



Divisibility by 6 : Can you identify a number which is divisible by both 2 and 3? One such number is 18. Will 18 be divisible by 2×3=6? Yes, it is.

Find some more numbers like 18 and check if they are divisible by 6 also.

Can you quickly think of a number which is divisible by 2 but not by 3?

Now for a number divisible by 3 but not by 2, one example is 27. Is 27 divisible by 6? No. Try to find numbers like 27.

From these observations we conclude that **if a number is divisible by 2 and 3 both then it is divisible by 6 also.**

Divisibility by 4 : Can you quickly give five 3-digit numbers divisible by 4? One such number is 212. Think of such 4-digit numbers. One example is 1936.

Observe the number formed by the ones and tens places of 212. It is 12; which is divisible by 4. For 1936 it is 36, again divisible by 4.

Try the exercise with other such numbers, for example with 4612; 3516; 9532.

Is the number 286 divisible by 4? No. Is 86 divisible by 4? No.

So, we see that **a number with 3 or more digits is divisible by 4 if the**

number formed by its last two digits (i.e. ones and tens) is divisible by 4. Check this rule by taking ten more examples.

Divisibility for 1 or 2 digit numbers by 4 has to be checked by actual division.

Divisibility by 8 : Are the numbers 1000, 2104, 1416 divisible by 8?

You can check that they are divisible by 8. Let us try to see the pattern.

Look at the digits at ones, tens and hundreds place of these numbers. These are 000, 104 and 416 respectively. These too are divisible by 8. Find some more numbers in which the number formed by the digits at units, tens and hundreds place (i.e. last 3 digits) is divisible by 8. For example, 9216, 8216, 7216, 10216, 9995216 etc. You will find that the numbers themselves are divisible by 8.

We find that **a number with 4 or more digits is divisible by 8, if the number formed by the last three digits is divisible by 8.**

Is 73512 divisible by 8?

The divisibility for numbers with 1, 2 or 3 digits by 8 has to be checked by actual division.

Divisibility by 9 : The multiples of 9 are 9, 18, 27, 36, 45, 54,... There are other numbers like 4608, 5283 that are also divisible by 9.

Do you find any pattern when the digits of these numbers are added?

1 + 8 = 9, 2 + 7 = 9, 3 + 6 = 9, 4 + 5 = 9

4 + 6 + 0 + 8 = 18, 5 + 2 + 8 + 3 = 18

All these sums are also divisible by 9.

Is the number 758 divisible by 9?

No. The sum of its digits 7 + 5 + 8 = 20 is also not divisible by 9.

These observations lead us to say that **if the sum of the digits of a number is divisible by 9, then the number itself is divisible by 9.**

Divisibility by 11 : The numbers 308, 1331 and 61809 are all divisible by 11. We form a table and see if the digits in these numbers lead us to some pattern.

Number	Sum of the digits (at odd places) from the right	Sum of the digits (at even places) from the right	Difference
308	8 + 3 = 11	0	11 – 0 = 11
1331	1 + 3 = 4	3 + 1 = 4	4 – 4 = 0
61809	9 + 8 + 6 = 23	0 + 1 = 1	23 – 1 = 22

We observe that in each case the difference is either 0 or divisible by 11. All these numbers are also divisible by 11.

For the number 5081, the difference of the digits is (5+8) – (1+0) = 12 which is not divisible by 11. The number 5081 is also not divisible by 11.

Thus, to check the divisibility of a number by 11, the rule is, **find the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number. If the difference is either 0 or divisible by 11, then the number is divisible by 11**.

EXERCISE 3.3

1. Using divisibility tests, determine which of the following numbers are divisible by 2; by 3; by 4; by 5; by 6; by 8; by 9; by 10 ; by 11 (say, yes or no):

Number	**Divisible by**								
	2	**3**	**4**	**5**	**6**	**8**	**9**	**10**	**11**
128	Yes	No	Yes	No	No	Yes	No	No	No
990	.....	.....	.....	.....	.....	.....	.....	.....	.....
1586	.....	.....	.....	.....	.....	.....	.....	.....	.....
275	.....	.....	.....	.....	.....	.....	.....	.....	.....
6686	.....	.....	.....	.....	.....	.....	.....	.....	.....
639210	.....	.....	.....	.....	.....	.....	.....	.....	.....
429714	.....	.....	.....	.....	.....	.....	.....	.....	.....
2856	.....	.....	.....	.....	.....	.....	.....	.....	.....
3060	.....	.....	.....	.....	.....	.....	.....	.....	.....
406839	.....	.....	.....	.....	.....	.....	.....	.....	.....

2. Using divisibility tests, determine which of the following numbers are divisible by 4; by 8:
   (a) 572 (b) 726352 (c) 5500 (d) 6000 (e) 12159
   (f) 14560 (g) 21084 (h) 31795072 (i) 1700 (j) 2150
3. Using divisibility tests, determine which of following numbers are divisible by 6:
   (a) 297144 (b) 1258 (c) 4335 (d) 61233 (e) 901352
   (f) 438750 (g) 1790184 (h) 12583 (i) 639210 (j) 17852
4. Using divisibility tests, determine which of the following numbers are divisible by 11:
   (a) 5445 (b) 10824 (c) 7138965 (d) 70169308 (e) 10000001
   (f) 901153
5. Write the smallest digit and the greatest digit in the blank space of each of the following numbers so that the number formed is divisible by 3 :
   (a) __ 6724 (b) 4765 __ 2

6. Write a digit in the blank space of each of the following numbers so that the number formed is divisible by 11 :
   (a) 92 __ 389 (b) 8 __ 9484

3.5 Common Factors and Common Multiples

Observe the factors of some numbers taken in pairs.

(a) What are the factors of 4 and 18?

The factors of 4 are 1, 2 and 4.

The factors of 18 are 1, 2, 3, 6, 9 and 18.

The numbers 1 and 2 are the factors of both 4 and 18.

They are the common factors of 4 and 18.

Try These

Find the common factors of
(a) 8, 20 (b) 9, 15

(b) What are the common factors of 4 and 15?

These two numbers have only 1 as the common factor.

What about 7 and 16?

Two numbers having only 1 as a common factor are called co-prime numbers. Thus, 4 and 15 are co-prime numbers.

Are 7 and 15, 12 and 49, 18 and 23 co-prime numbers?

(c) Can we find the common factors of 4, 12 and 16?
Factors of 4 are 1, 2 and 4.
Factors of 12 are 1, 2, 3, 4, 6 and 12.
Factors of 16 are 1, 2, 4, 8 and 16.
Clearly, 1, 2 and 4 are the common factors of 4, 12, and 16.
Find the common factors of (a) 8, 12, 20 (b) 9, 15, 21.

Let us now look at the multiples of more than one number taken at a time.

(a) What are the multiples of 4 and 6?

The multiples of 4 are 4, 8, 12, 16, 20, 24, ... (write a few more)

The multiples of 6 are 6, 12, 18, 24, 30, 36, ... (write a few more)

Out of these, are there any numbers which occur in both the lists?

We observe that 12, 24, 36, ... are multiples of both 4 and 6.
Can you write a few more?
They are called the common multiples of 4 and 6.

(b) Find the common multiples of 3, 5 and 6.
Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, ...
Multiples of 5 are 5, 10, 15, 20, 25, 30, 35, ...
Multiples of 6 are 6, 12, 18, 24, 30, ...
Common multiples of 3, 5 and 6 are 30, 60, ...

Write a few more common multiples of 3, 5 and 6.

Example 5 : Find the common factors of 75, 60 and 210.

Solution : Factors of 75 are 1, 3, 5, 15, 25 and 75.

Factors of 60 are 1, 2, 3, 4, 5, 6, 10, 12, 15, 30 and 60.

Factors of 210 are 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105 and 210.

Thus, common factors of 75, 60 and 210 are 1, 3, 5 and 15.

Example 6 : Find the common multiples of 3, 4 and 9.

Solution : Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, ...

Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48,...

Multiples of 9 are 9, 18, 27, 36, 45, 54, 63, 72, 81, ...

Clearly, common multiples of 3, 4 and 9 are 36, 72, 108,...

EXERCISE 3.4

1. Find the common factors of :
   (a) 20 and 28 (b) 15 and 25 (c) 35 and 50 (d) 56 and 120
2. Find the common factors of :
   (a) 4, 8 and 12 (b) 5, 15 and 25
3. Find first three common multiples of :
   (a) 6 and 8 (b) 12 and 18
4. Write all the numbers less than 100 which are common multiples of 3 and 4.
5. Which of the following numbers are co-prime?
   (a) 18 and 35 (b) 15 and 37 (c) 30 and 415
   (d) 17 and 68 (e) 216 and 215 (f) 81 and 16
6. A number is divisible by both 5 and 12. By which other number will that number be always divisible?
7. A number is divisible by 12. By what other numbers will that number be divisible?

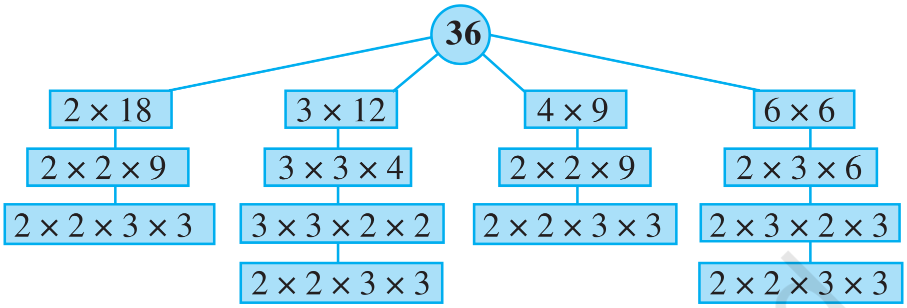
3.6 Prime Factorisation

When a number is expressed as a product of its factors we say that the number has been factorised. Thus, when we write 24 = 3×8, we say that 24 has been factorised. This is one of the factorisations of 24. The others are :

$24 = 2 \times 12$	$24 = 4 \times 6$	$24 = 3 \times 8$
$= 2 \times 2 \times 6$	$= 2 \times 2 \times 6$	$= 3 \times 2 \times 2 \times 2$
$= 2 \times 2 \times 2 \times 3$	$= 2 \times 2 \times 2 \times 3$	$= 2 \times 2 \times 2 \times 3$

In all the above factorisations of 24, we ultimately arrive at only one factorisation 2 × 2 × 2 × 3. In this factorisation the only factors 2 and 3 are prime numbers. Such a factorisation of a number is called a *prime factorisation*.

Let us check this for the number 36.



The prime factorisation of 36 is 2 × 2 × 3 × 3. i.e. the only prime factorisation of 36.

Do This

Factor tree

Choose a number and write it

Think of a factor pair say, 90=10×9

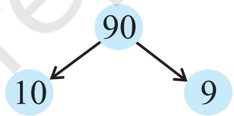
90

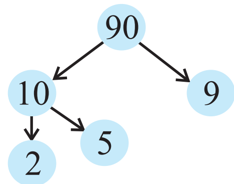
Now think of a factor pair of 10

10 = 2×5

Write factor pair of 9

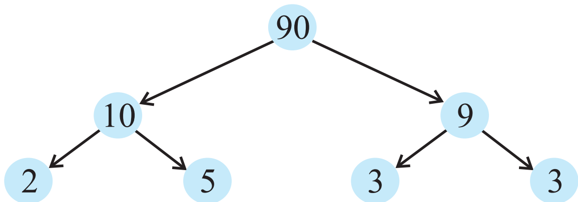
9 = 3 × 3





Try this for the numbers

(a) 8 (b) 12



Try These

Write the prime factorisations of 16, 28, 38.

Example 7 : Find the prime factorisation of 980.

Solution : We proceed as follows:

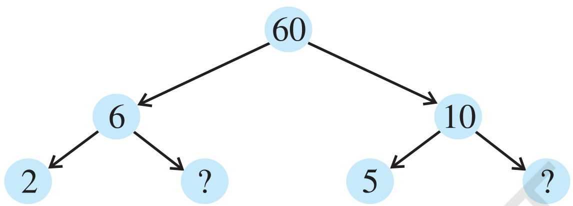
We divide the number 980 by 2, 3, 5, 7 etc. in this order repeatedly so long as the quotient is divisible by that number.Thus, the prime factorisation of 980 is 2 × 2 × 5 × 7 × 7.

2	980
2	490
5	245
7	49
7	7
	1

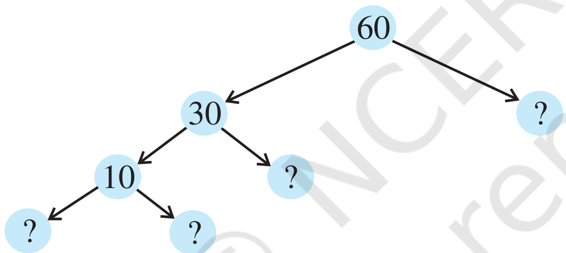
EXERCISE 3.5

1. Here are two different factor trees for 60. Write the missing numbers.

   (a)



   (b)



2. Which factors are not included in the prime factorisation of a composite number?
3. Write the greatest 4-digit number and express it in terms of its prime factors.
4. Write the smallest 5-digit number and express it in the form of its prime factors.
5. Find all the prime factors of 1729 and arrange them in ascending order. Now state the relation, if any; between two consecutive prime factors.
6. The product of three consecutive numbers is always divisible by 6. Verify this statement with the help of some examples.
7. The sum of two consecutive odd numbers is divisible by 4. Verify this statement with the help of some examples.
8. In which of the following expressions, prime factorisation has been done?

   (a) $24 = 2 \times 3 \times 4$ (b) $56 = 7 \times 2 \times 2 \times 2$

   (c) $70 = 2 \times 5 \times 7$ (d) $54 = 2 \times 3 \times 9$
9. 18 is divisible by both 2 and 3. It is also divisible by $2 \times 3 = 6$. Similarly, a number is divisible by both 4 and 6. Can we say that the number must also be divisible by $4 \times 6 = 24$? If not, give an example to justify your answer.
10. I am the smallest number, having four different prime factors. Can you find me?

3.7 Highest Common Factor

We can find the common factors of any two numbers. We now try to find the highest of these common factors.

What are the common factors of 12 and 16? They are 1, 2 and 4.

What is the highest of these common factors? It is 4.

What are the common factors of 20, 28 and 36? They are 1, 2 and 4 and again 4 is highest of these common factors.

Try These

Find the HCF of the following:

(i) 24 and 36 (ii) 15, 25 and 30

(iii) 8 and 12 (iv) 12, 16 and 28

The Highest Common Factor (HCF) of two or more given numbers is the highest (or greatest) of their common factors. It is also known as Greatest Common Divisor (GCD).

The HCF of 20, 28 and 36 can also be found by prime factorisation of these numbers as follows:

$$\begin{array}{r|l} 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 28 \\ \hline 2 & 14 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Thus,

$$20 = \boxed{2} \times \boxed{2} \times 5$$
$$28 = \boxed{2} \times \boxed{2} \times 7$$
$$36 = \boxed{2} \times \boxed{2} \times 3 \times 3$$

The common factor of 20, 28 and 36 is 2(occuring twice). Thus, HCF of 20, 28 and 36 is $2 \times 2 = 4$.

EXERCISE 3.6

1. Find the HCF of the following numbers :

   (a) 18, 48 (b) 30, 42 (c) 18, 60 (d) 27, 63

   (e) 36, 84 (f) 34, 102 (g) 70, 105, 175

   (h) 91, 112, 49 (i) 18, 54, 81 (j) 12, 45, 75

2. What is the HCF of two consecutive

   (a) numbers? (b) even numbers? (c) odd numbers?

3. HCF of co-prime numbers 4 and 15 was found as follows by factorisation : $4 = 2 \times 2$ and $15 = 3 \times 5$ since there is no common prime factor, so HCF of 4 and 15 is 0. Is the answer correct? If not, what is the correct HCF?

3.8 Lowest Common Multiple

What are the common multiples of 4 and 6? They are 12, 24, 36, ... Which is the lowest of these? It is 12. We say that lowest common multiple of 4 and 6 is 12. It is the smallest number that both the numbers are factors of this number.

The Lowest Common Multiple (LCM) of two or more given numbers is the lowest (or smallest or least) of their common multiples.

What will be the LCM of 8 and 12? 4 and 9? 6 and 9?

Example 8 : Find the LCM of 12 and 18.

Solution : We know that common multiples of 12 and 18 are 36, 72, 108 etc. The lowest of these is 36. Let us see another method to find LCM of two numbers.

The prime factorisations of 12 and 18 are :

$12 = 2 \times 2 \times 3; \quad 18 = 2 \times 3 \times 3$

In these prime factorisations, the maximum number of times the prime factor 2 occurs is two; this happens for 12. Similarly, the maximum number of times the factor 3 occurs is two; this happens for 18. The LCM of the two numbers is the product of the prime factors counted the maximum number of times they occur in any of the numbers. Thus, in this case LCM $= 2 \times 2 \times 3 \times 3 = 36$.

Example 9 : Find the LCM of 24 and 90.

Solution : The prime factorisations of 24 and 90 are:

$24 = 2 \times 2 \times 2 \times 3; \quad 90 = 2 \times 3 \times 3 \times 5$

In these prime factorisations the maximum number of times the prime factor 2 occurs is three; this happens for 24. Similarly, the maximum number of times the prime factor 3 occurs is two; this happens for 90. The prime factor 5 occurs only once in 90.

Thus, LCM $= (2 \times 2 \times 2) \times (3 \times 3) \times 5 = 360$

Example 10 : Find the LCM of 40, 48 and 45.

Solution : The prime factorisations of 40, 48 and 45 are;

$40 = 2 \times 2 \times 2 \times 5$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$45 = 3 \times 3 \times 5$

The prime factor 2 appears maximum number of four times in the prime factorisation of 48, the prime factor 3 occurs maximum number of two times in the prime factorisation of 45, The prime factor 5 appears one time in the prime factorisations of 40 and 45, we take it only once.

Therefore, required LCM = $(2 \times 2 \times 2 \times 2) \times (3 \times 3) \times 5 = 720$

LCM can also be found in the following way :

Example 11 : Find the LCM of 20, 25 and 30.

Solution : We write the numbers as follows in a row :

2	20	25	30	**(A)**
2	10	25	15	**(B)**
3	5	25	15	**(C)**
5	5	25	5	**(D)**
5	1	5	1	**(E)**
	1	1	1	

So, LCM = $2 \times 2 \times 3 \times 5 \times 5$.

(A) Divide by the least prime number which divides atleast one of the given numbers. Here, it is 2. The numbers like 25 are not divisible by 2 so they are written as such in the next row.

(B) Again divide by 2. Continue this till we have no multiples of 2.

(C) Divide by next prime number which is 3.

(D) Divide by next prime number which is 5.

(E) Again divide by 5.

3.9 Some Problems on HCF and LCM

We come across a number of situations in which we make use of the concepts of HCF and LCM. We explain these situations through a few examples.

Example 12 : Two tankers contain 850 litres and 680 litres of kerosene oil respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

Solution : The required container has to measure both the tankers in a way that the count is an exact number of times. So its capacity must be an exact divisor of the capacities of both the tankers. Moreover, this capacity should be **maximum**. Thus, the maximum capacity of such a container will be the HCF of 850 and 680.

It is found as follows :

2	850
5	425
5	85
17	17
	1

2	680
2	340
2	170
5	85
17	17
	1

Hence,

$850 = 2 \times 5 \times 5 \times 17 \quad = \boxed{2} \times \boxed{5} \times \boxed{17} \times 5$ and

$680 = 2 \times 2 \times 2 \times 5 \times 17 \quad = \boxed{2} \times \boxed{5} \times \boxed{17} \times 2 \times 2$

The common factors of 850 and 680 are 2, 5 and 17.

Thus, the HCF of 850 and 680 is $2 \times 5 \times 17 = 170$.

Therefore, maximum capacity of the required container is 170 litres.

It will fill the first container in 5 and the second in 4 refills.

Example 13 : In a morning walk, three persons step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

Solution : The distance covered by each one of them is required to be the same as well as minimum. The required minimum distance each should walk would be the lowest common multiple of the measures of their steps. Can you describe why? Thus, we find the LCM of 80, 85 and 90. The LCM of 80, 85 and 90 is 12240.

The required minimum distance is 12240 cm.

Example 14 : Find the least number which when divided by 12, 16, 24 and 36 leaves a remainder 7 in each case.

Solution : We first find the LCM of 12, 16, 24 and 36 as follows :

2	12	16	24	36
2	6	8	12	18
2	3	4	6	9
2	3	2	3	9
3	3	1	3	9
3	1	1	1	3
	1	1	1	1

Thus, LCM = $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

144 is the least number which when divided by the given numbers will leave remainder 0 in each case. But we need the least number that leaves remainder 7 in each case.

Therefore, the required number is 7 more than 144. The required least number = 144 + 7 = 151.

EXERCISE 3.7

1. Renu purchases two bags of fertiliser of weights 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the fertiliser exact number of times.
2. Three boys step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?
3. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.
4. Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12.
5. Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.
6. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?
7. Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a container that can measure the diesel of the three containers exact number of times.
8. Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.
9. Find the smallest 4-digit number which is divisible by 18, 24 and 32.
10. Find the LCM of the following numbers :

    (a) 9 and 4 (b) 12 and 5 (c) 6 and 5 (d) 15 and 4

    Observe a common property in the obtained LCMs. Is LCM the product of two numbers in each case?
11. Find the LCM of the following numbers in which one number is the factor of the other.

    (a) 5, 20 (b) 6, 18 (c) 12, 48 (d) 9, 45

    What do you observe in the results obtained?

What have we discussed?

1. We have discussed multiples, divisors, factors and have seen how to identify factors and multiples.
2. We have discussed and discovered the following :
   (a) A factor of a number is an exact divisor of that number.
   (b) Every number is a factor of itself. 1 is a factor of every number.
   (c) Every factor of a number is less than or equal to the given number.
   (d) Every number is a multiple of each of its factors.
   (e) Every multiple of a given number is greater than or equal to that number.
   (f) Every number is a multiple of itself.
3. We have learnt that –
   (a) The number other than 1, with only factors namely 1 and the number itself, is a prime number. Numbers that have more than two factors are called composite numbers. Number 1 is neither prime nor composite.
   (b) The number 2 is the smallest prime number and is even. Every prime number other than 2 is odd.
   (c) Two numbers with only 1 as a common factor are called co-prime numbers.
   (d) A number divisible by two co-prime numbers is divisible by their product also.
4. We have discussed how we can find just by looking at a number, whether it is divisible by small numbers 2,3,4,5,8,9 and 11. We have explored the relationship between digits of the numbers and their divisibility by different numbers.
   (a) Divisibility by 2,5 and 10 can be seen by just the last digit.
   (b) Divisibility by 3 and 9 is checked by finding the sum of all digits.
   (c) Divisibility by 4 and 8 is checked by the last 2 and 3 digits respectively.
   (d) Divisibility of 11 is checked by comparing the sum of digits at odd and even places.
5. We have learnt that –
   (a) The Highest Common Factor (HCF) of two or more given numbers is the highest of their common factors.
   (b) The Lowest Common Multiple (LCM) of two or more given numbers is the lowest of their common multiples.

Basic Geometrical Ideas

Chapter 4

0650CH04

4.1 Introduction

Geometry has a long and rich history. The term 'Geometry' is the English equivalent of the Greek word '*Geometron*'. '*Geo*' means Earth and '*metron*' means Measurement. According to historians, the geometrical ideas shaped up in ancient times, probably due to the need in art, architecture and measurement. These include occasions when the boundaries of cultivated lands had to be marked without giving room for complaints. Construction of magnificent palaces, temples, lakes, dams and cities, art and architecture propped up these ideas. Even today geometrical ideas are reflected in all forms of art, measurements, architecture, engineering, cloth designing etc. You observe and use different objects like boxes, tables, books, the tiffin box you carry to your school for lunch, the ball with which you play and so on. All such objects have different shapes. The ruler which you use, the pencil with which you write are straight. The pictures of a bangle, the one rupee coin or a ball appear round.

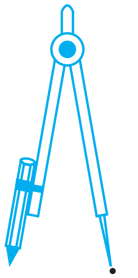
Here, you will learn some interesting facts that will help you know more about the shapes around you.

4.2 Points

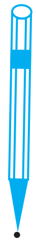
By a sharp tip of the pencil, mark a dot on the paper. Sharper the tip, thinner will be the dot. This almost invisible tiny dot will give you an idea of a point.

A point determines a location.

These are some models for a point :



The tip of a compass



The sharpened end of a pencil

The pointed end of a needle

If you mark three points on a paper, you would be required to distinguish them. For this they are denoted by a single capital letter like A,B,C.

•B

•A

•C

These points will be read as point A, point B and point C.

Of course, the dots have to be invisibly thin.

Try These

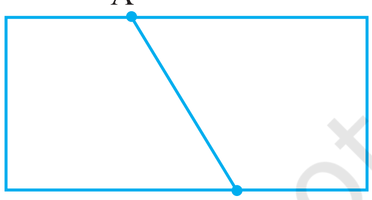
1. With a sharp tip of the pencil, mark four points on a paper and name them by the letters A,C,P,H. Try to name these points in different ways. One such way could be this

   A• •C

   P• •H

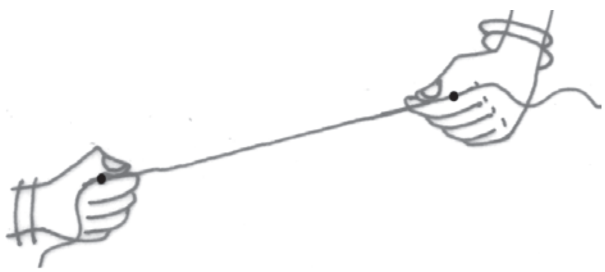
2. A star in the sky also gives us an idea of a point. Identify at least five such situations in your daily life.

4.3 A Line Segment

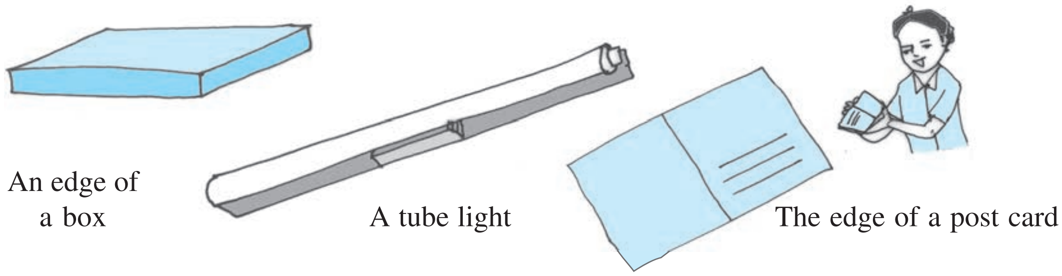


Fold a piece of paper and unfold it. Do you see a fold? This gives the idea of a line segment. It has two end points A and B.

Take a thin thread. Hold its two ends and stretch it without a slack. It represents a line segment. The ends held by hands are the end points of the line segment.



The following are some models for a line segment :



Try to find more examples for line segments from your surroundings.

Mark any two points A and B on a sheet of paper. Try to connect A to B by all possible routes. (Fig 4.1)

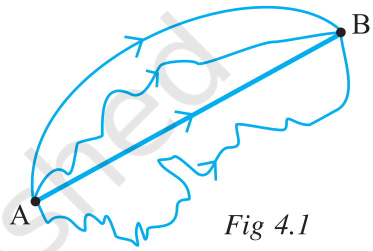


Fig 4.1

What is the shortest route from A to B?

This shortest join of point A to B (including A and B) shown here is a line segment. It is denoted by $\overline{AB}$ or $\overline{BA}$. The points A and B are called the end points of the segment.

Try These

1. Name the line segments in the figure 4.2.
   Is A, the end point of each line segment?

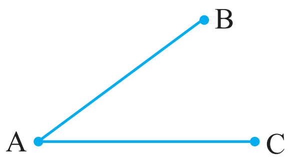


Fig 4.2

4.4 A Line

Imagine that the line segment from A to B (i.e. $\overline{AB}$) is extended beyond A in one direction and beyond B in the other direction without any end (see figure). You now get a **model for a line**.

Do you think you can draw a complete picture of a line? No. (Why?)

A line through two points A and B is written as $\overleftrightarrow{AB}$. It extends indefinitely in both directions. So it contains a countless number of points. (Think about this).

Two points are enough to fix a line. We say 'two points determine a line'.

The adjacent diagram (Fig 4.3) is that of a line PQ written as $\overleftrightarrow{PQ}$. Sometimes a line is denoted by a letter like *l*, *m*.

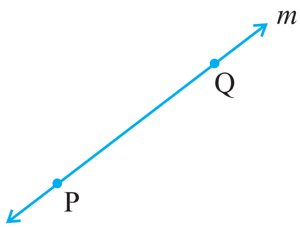


Fig 4.3

4.5 Intersecting Lines

Look at the diagram (Fig 4.4). Two lines l_1 and l_2 are shown. Both the lines pass through point P. We say l_1 and l_2 intersect at P. If two lines have one common point, they are called *intersecting lines*.

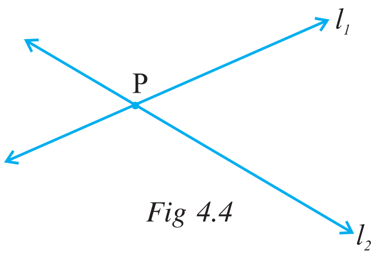
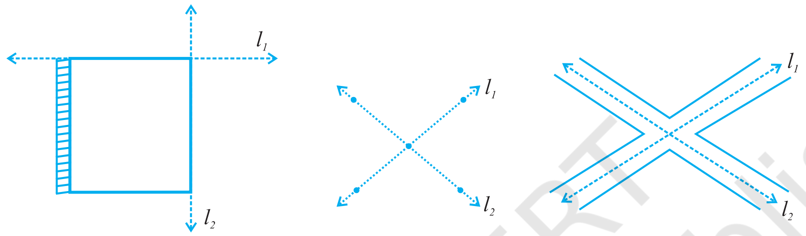


Fig 4.4

The following are some models of a pair of intersecting lines (Fig 4.5) :

Try to find out some more models for a pair of intersecting lines.



Two adjacement edges of your notebook | The letter X of the English alphabet | Crossing-roads

Fig 4.5

Do This

Take a sheet of paper. Make two folds (and crease them) to represent a pair of intersecting lines and discuss :

(a) Can two lines intersect in more than one point?

(b) Can more than two lines intersect in one point?

4.6 Parallel Lines

Let us look at this table (Fig 4.6). The top ABCD is flat. Are you able to see some points and line segments?

Are there intersecting line segments?

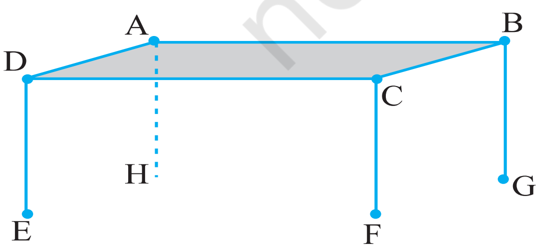


Fig 4.6

Yes, AB and BC intersect at the point B.

Which line segments intersect at A? at C? at D?

Do the lines AD and CD intersect?

Do the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ intersect?

You find that on the table's surface there are line segment which will not meet, however far they are extended. $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ form one such pair. Can you identify one more such pair of lines (which do not meet) on the top of the table?

Lines like these which do not meet are said to be parallel; and are called **parallel lines**.

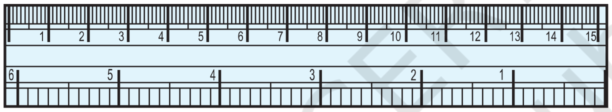
Think, discuss and write

Where else do you see parallel lines? Try to find ten examples.

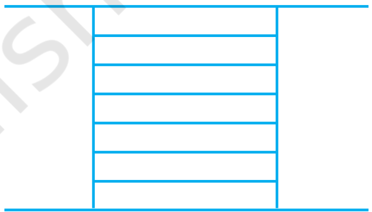
If two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel, we write $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

If two lines l_1 and l_2 are parallel, we write $l_1 \parallel l_2$.

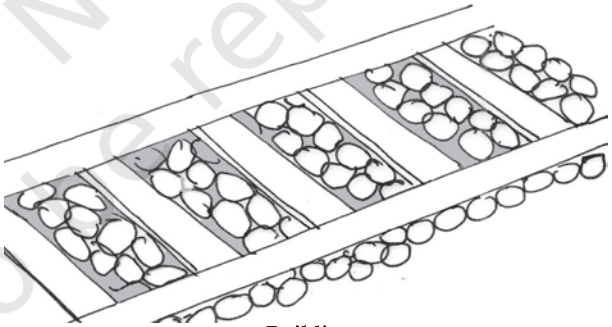
Can you identify parrallel lines in the following figures?



The opposite edges of ruler (scale)

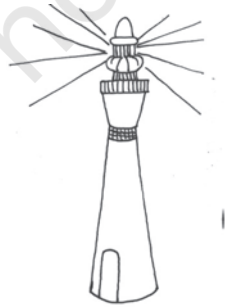


The cross-bars of this window

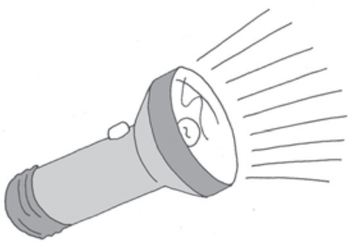


Rail lines

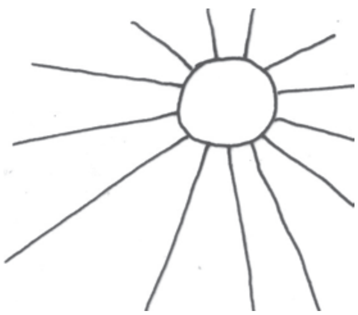
4.7 Ray



Beam of light from a light house



Ray of light from a torch



Sun rays

The following are some models for a ray :

A *ray* is a portion of a line. It starts at one point (called starting point or initial point) and goes endlessly in a direction.

Look at the diagram (Fig 4.7) of ray shown here. Two points are shown on the ray. They are (a) A, the starting point (b) P, a point on the path of the ray.

We denote it by $\overrightarrow{AP}$.

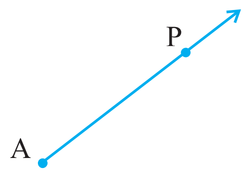


Fig 4.7

Think, discuss and write

If $\overrightarrow{PQ}$ is a ray,

(a) What is its starting point?

(b) Where does the point Q lie on the ray?

(c) Can we say that Q is the starting point of this ray?

Try These

1. Name the rays given in this picture (Fig 4.8).
2. Is T a starting point of each of these rays?

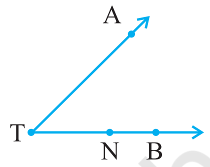


Fig 4.8

Here is a ray $\overrightarrow{OA}$ (Fig 4.9). It starts at O and passes through the point A. It also passes through the point B.

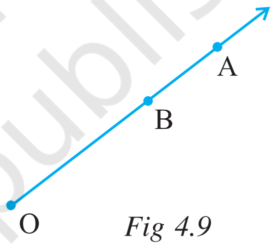


Fig 4.9

Can you also name it as $\overrightarrow{OB}$? Why?

$\overrightarrow{OA}$ and $\overrightarrow{OB}$ are same here.

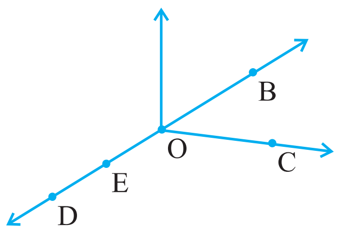
Can we write $\overrightarrow{OA}$ as $\overrightarrow{AO}$? Why or why not?

Draw five rays and write appropriate names for them.

What do the arrows on each of these rays show?

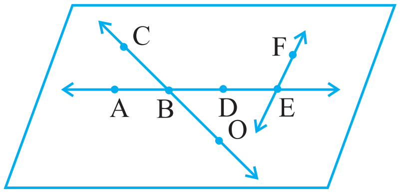
EXERCISE 4.1

1. Use the figure to name :
   (a) Five points
   (b) A line
   (c) Four rays
   (d) Five line segments

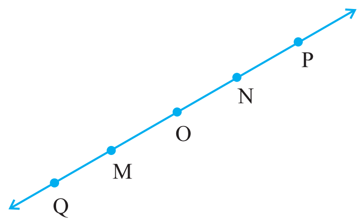


2. Name the line given in all possible (twelve) ways, choosing only two letters at a time from the four given.

3. Use the figure to name :
   (a) Line containing point E.
   (b) Line passing through A.
   (c) Line on which O lies
   (d) Two pairs of intersecting lines.



4. How many lines can pass through (a) one given point? (b) two given points?
5. Draw a rough figure and label suitably in each of the following cases:
   (a) Point P lies on $\overline{AB}$.
   (b) $\overleftrightarrow{XY}$ and $\overleftrightarrow{PQ}$ intersect at M.
   (c) Line *l* contains E and F but not D.
   (d) $\overleftrightarrow{OP}$ and $\overleftrightarrow{OQ}$ meet at O.
6. Consider the following figure of line $\overleftrightarrow{MN}$. Say whether following statements are true or false in context of the given figure.
   (a) Q, M, O, N, P are points on the line $\overleftrightarrow{MN}$.
   (b) M, O, N are points on a line segment $\overline{MN}$.
   (c) M and N are end points of line segment $\overline{MN}$.
   (d) O and N are end points of line segment $\overline{OP}$.
   (e) M is one of the end points of line segment $\overline{QO}$.
   (f) M is point on ray $\overrightarrow{OP}$.
   (g) Ray $\overrightarrow{OP}$ is different from ray $\overrightarrow{QP}$.
   (h) Ray $\overrightarrow{OP}$ is same as ray $\overrightarrow{OM}$.
   (i) Ray $\overrightarrow{OM}$ is not opposite to ray $\overrightarrow{OP}$.
   (j) O is not an initial point of $\overrightarrow{OP}$.
   (k) N is the initial point of $\overrightarrow{NP}$ and $\overrightarrow{NM}$.



4.8 Curves

Have you ever taken a piece of paper and just doodled? The pictures that are results of your doodling are called *curves.*

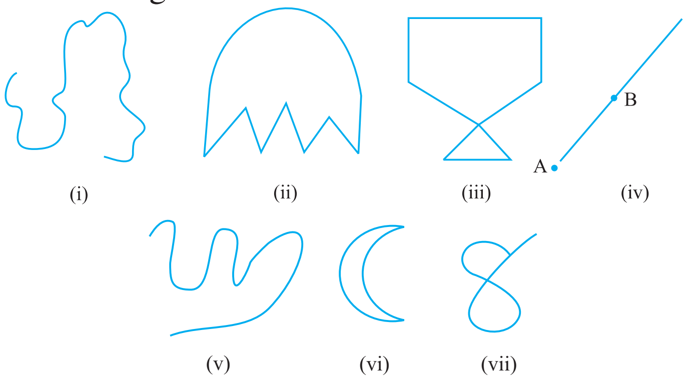


Fig 4.10

You can draw some of these drawings without lifting the pencil from the paper and without the use of a ruler. These are all curves (Fig 4.10).

'Curve' in everyday usage means "not straight". In Mathematics, a curve can be straight like the one shown in fig 4.10 (iv).

Observe that the curves (iii) and (vii) in Fig 4.10 cross themselves, whereas the curves (i), (ii), (v) and (vi) in Fig 4.10 do not. If a curve does not cross itself, then it is called a **simple curve**.

Draw five more simple curves and five curves that are not simple.

Consider these now (Fig 4.11).

What is the difference between these two? The first i.e. Fig 4.11 (i) is an **open curve** and the second i.e. Fig 4.11(ii) is a **closed curve**. Can you identify some closed and open curves from the figures Fig 4.10 (i), (ii), (v), (vi)? Draw five curves each that are open and closed.

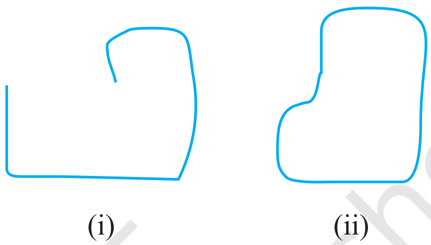


Fig 4.11

Position in a figure

A court line in a tennis court divides it into three parts : inside the line, on the line and outside the line. You cannot enter inside without crossing the line.

A compound wall separates your house from the road. You talk about 'inside' the compound, 'on' the boundary of the compound and 'outside' the compound.

In a closed curve, thus, there are three parts.

(i) interior ('inside') of the curve

(ii) boundary ('on') of the curve and

(iii) exterior ('outside') of the curve.

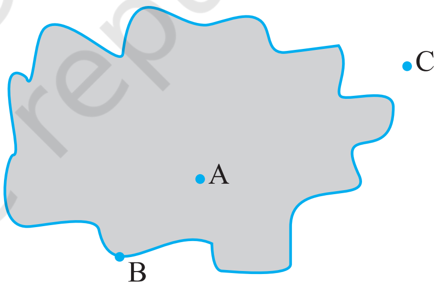


Fig 4.12

In the figure 4.12, A is in the interior, C is in the exterior and B is on the curve.

The interior of a curve together with its boundary is called its "**region**".

4.9 Polygons

Look at these figures 4.13 (i), (ii), (iii), (iv), (v) and (vi).

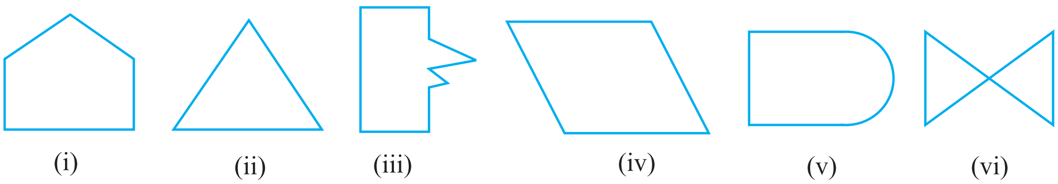


Fig 4.13

What can you say? Are they closed? How does each one of them differ from the other? (i), (ii), (iii), (iv) and (vi) are special because they are made up entirely of line segments. Out of these (i), (ii), (iii) and (iv) are also simple closed curves. They are called **polygons.**

So, a figure is a polygon if it is a simple closed figure made up entirely of line segments. Draw ten differently shaped polygons.

Do This

Try to form a polygon with

1. Five matchsticks.
2. Four matchsticks.
3. Three matchsticks.
4. Two matchsticks.

In which case was it not possible? Why?

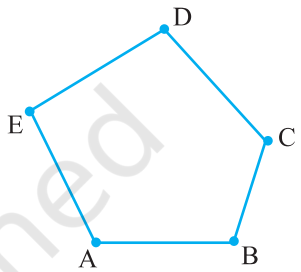


Fig 4.14

Sides, vertices and diagonals

Examine the figure given here (Fig 4.14).

Give justification to call it a polygon.

The line segments forming a polygon are called its *sides*.

What are the sides of polygon ABCDE? (Note how the corners are named in order.)

Sides are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$ and $\overline{EA}$.

The meeting point of a pair of sides is called its *vertex*.

Sides $\overline{AE}$ and $\overline{ED}$ meet at E, so E is a vertex of the polygon ABCDE. Points B and C are its other vertices. Can you name the sides that meet at these points?

Can you name the other vertices of the above polygon ABCDE?

Any two sides with a common end point are called the *adjacent sides* of the polygon.

Are the sides $\overline{AB}$ and $\overline{BC}$ adjacent? How about $\overline{AE}$ and $\overline{DC}$?

The end points of the same side of a polygon are called the *adjacent vertices*. Vertices E and D are adjacent, whereas vertices A and D are not adjacent vertices. Do you see why?

Consider the pairs of vertices which are not adjacent. The joins of these vertices are called the diagonals of the polygon.

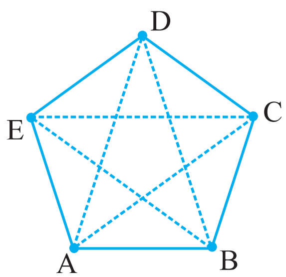


Fig 4.15

In the figure 4.15, $\overline{AC}$, $\overline{AD}$, $\overline{BD}$, $\overline{BE}$ and $\overline{CE}$ are diagonals.

Is $\overline{BC}$ a diagonal, Why or why not?

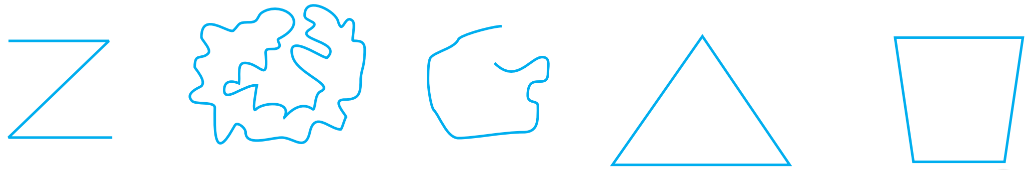
If you try to join adjacent vertices, will the result be a diagonal?

Name all the sides, adjacent sides, adjacent vertices of the figure ABCDE (Fig 4.15).

Draw a polygon ABCDEFGH and name all the sides, adjacent sides and vertices as well as the diagonals of the polygon.

EXERCISE 4.2

1. Classify the following curves as (i) Open or (ii) Closed.



(a) (b) (c) (d) (e)

2. Draw rough diagrams to illustrate the following :
   (a) Open curve (b) Closed curve.
3. Draw any polygon and shade its interior.
4. Consider the given figure and answer the questions :
   (a) Is it a curve? (b) Is it closed?

5. Illustrate, if possible, each one of the following with a rough diagram:
   (a) A closed curve that is not a polygon.
   (b) An open curve made up entirely of line segments.
   (c) A polygon with two sides.

4.10 Angles

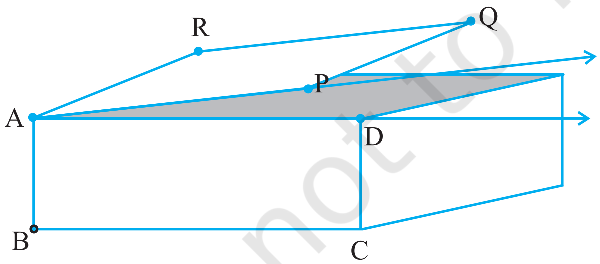


Fig 4.16

Angles are made when corners are formed.

Here is a picture (Fig 4.16) where the top of a box is like a hinged lid. The edges AD of the box and AP of the door can be imagined as two rays $\overrightarrow{AD}$ and $\overrightarrow{AP}$. These two rays have a common initial point A. The two rays here together are said to form an angle.

An angle is made up of two rays starting from a common initial point.

The two rays forming the angle are called the *arms* or *sides* of the angle.

The common initial point is the *vertex* of the angle.

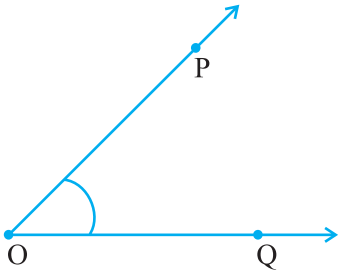


Fig 4.17

This is an angle formed by rays $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ (Fig 4.17). To show this we use a small curve at the vertex. (see Fig 4.17). O is the vertex. What are the sides? Are they not $\overrightarrow{OP}$ and $\overrightarrow{OQ}$?

How can we name this angle? We can simply say that it is an angle at O. To be more specific we identify some two points, one on each side and the vertex to name the angle. Angle POQ is thus a better way of naming the angle. We denote this by $\angle POQ$.

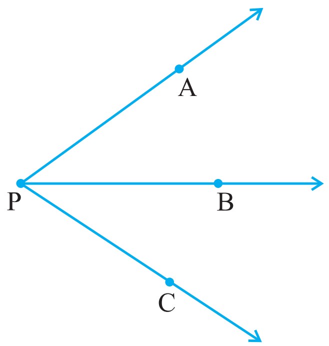


Fig 4.18

Think, discuss and write

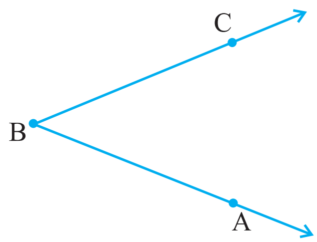
Look at the diagram (Fig 4.18).What is the name of the angle? Shall we say $\angle P$? But then which one do we mean? By $\angle P$ what do we mean? Is naming an angle by vertex helpful here? Why not?

By $\angle P$ we may mean $\angle APB$ or $\angle CPB$ or even $\angle APC$! We need more information.

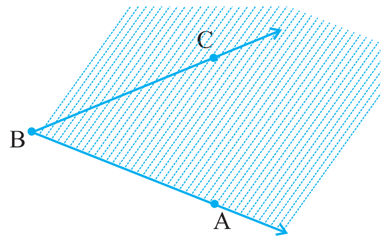
Note that in specifying the angle, the vertex is always written as the middle letter.

Do This

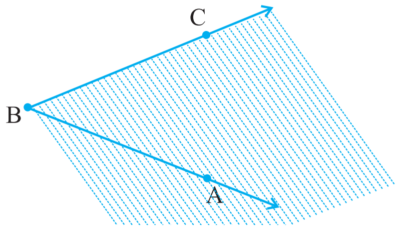
Take any angle, say $\angle ABC$.



Shade that portion of the paper bordering $\overrightarrow{BA}$ and where $\overrightarrow{BC}$ lies.



Now shade in a different colour the portion of the paper bordering $\overrightarrow{BC}$ and where $\overrightarrow{BA}$ lies.



The portion common to both shadings is called the interior of $\angle ABC$ (Fig 4.19). (Note that **the interior** is not a restricted area; it extends indefinitely since the two sides extend indefinitely).

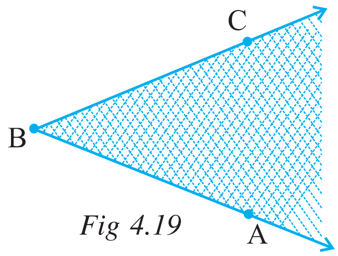


Fig 4.19

In this diagram (Fig 4.20), X is in the interior of the angle, Z is not in the interior but in the exterior of the angle; and S is on the $\angle PQR$. Thus, the angle also has three parts associated with it.

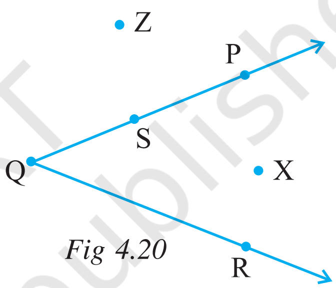
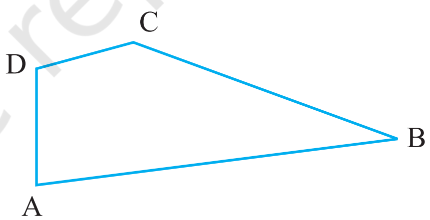


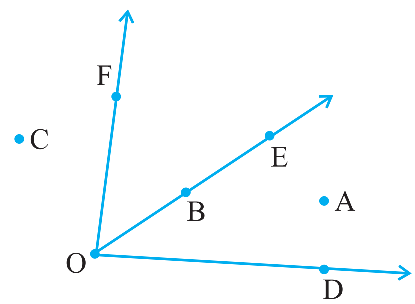
Fig 4.20

EXERCISE 4.3

1. Name the angles in the given figure.



2. In the given diagram, name the point(s)
   (a) In the interior of $\angle DOE$
   (b) In the exterior of $\angle EOF$
   (c) On $\angle EOF$



3. Draw rough diagrams of two angles such that they have
   (a) One point in common.
   (b) Two points in common.
   (c) Three points in common.
   (d) Four points in common.
   (e) One ray in common.

What have we discussed?

1. A point determines a location. It is usually denoted by a capital letter.
2. A line segment corresponds to the shortest distance between two points. The line segment joining points A and B is denoted by $\overline{AB}$.
3. A line is obtained when a line segment like $\overline{AB}$ is extended on both sides indefinitely; it is denoted by $\overleftrightarrow{AB}$ or sometimes by a single small letter like *l*.
4. Two distinct lines meeting at a point are called *intersecting lines*.
5. Two lines in a plane are said to be parallel if they do not meet.
6. A ray is a portion of line starting at a point and going in one direction endlessly.
7. Any drawing (straight or non-straight) done without lifting the pencil may be called a curve. In this sense, a line is also a curve.
8. A simple curve is one that does not cross itself.
9. A curve is said to be closed if its ends are joined; otherwise it is said to be open.
10. A polygon is a simple closed curve made up of line segments. Here,
    (i) The line segments are the sides of the polygon.
    (ii) Any two sides with a common end point are adjacent sides.
    (iii) The meeting point of a pair of sides is called a *vertex*.
    (iv) The end points of the same side are adjacent vertices.
    (v) The join of any two non-adjacent vertices is a diagonal.
11. An angle is made up of two rays starting from a common starting/initial point.

    Two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ make $\angle AOB$ (or also called $\angle BOA$).
    An angle leads to three divisions of a region:
    On the angle, the interior of the angle and the exterior of the angle.

Understanding Elementary Shapes

0650CH05

Chapter 5

5.1 Introduction

All the shapes we see around us are formed using curves or lines. We can see corners, edges, planes, open curves and closed curves in our surroundings. We organise them into line segments, angles, triangles, polygons and circles. We find that they have different sizes and measures. Let us now try to develop tools to compare their sizes.

5.2 Measuring Line Segments

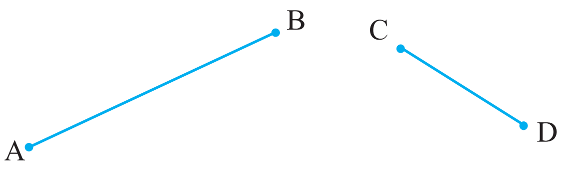
We have drawn and seen so many line segments. A triangle is made of three, a quadrilateral of four line segments.

A line segment is a fixed portion of a line. This makes it possible to measure a line segment. This measure of each line segment is a unique number called its "length". We use this idea to compare line segments.

To compare any two line segments, we find a relation between their lengths. This can be done in several ways.

(i) Comparison by observation:

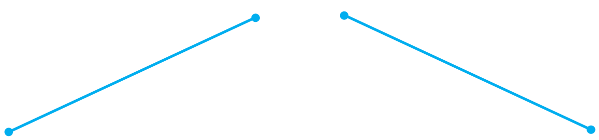
By just looking at them can you tell which one is longer?



You can see that $\overline{AB}$ is longer.

But you cannot always be sure about your usual judgment.

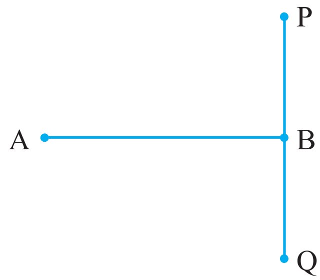
For example, look at the adjoining segments :



The difference in lengths between these two may not be obvious. This makes other ways of comparing necessary.

In this adjacent figure, $\overline{AB}$ and $\overline{PQ}$ have the same lengths. This is not quite obvious.

So, we need better methods of comparing line segments.



(ii) Comparison by Tracing

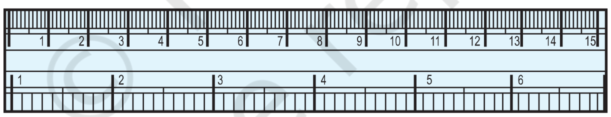
To compare $\overline{AB}$ and $\overline{CD}$, we use a tracing paper, trace $\overline{CD}$ and place the traced segment on $\overline{AB}$.

Can you decide now which one among $\overline{AB}$ and $\overline{CD}$ is longer?

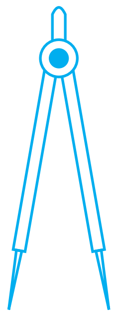
The method depends upon the accuracy in tracing the line segment. Moreover, if you want to compare with another length, you have to trace another line segment. This is difficult and you cannot trace the lengths everytime you want to compare them.

(iii) Comparison using Ruler and a Divider

Have you seen or can you recognise all the instruments in your instrument box? Among other things, you have a ruler and a divider.



Ruler

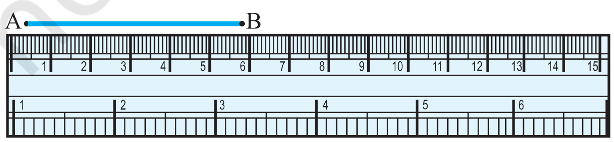


Divider

Note how the ruler is marked along one of its edges. It is divided into 15 parts. Each of these 15 parts is of length 1cm.

Each centimetre is divided into 10subparts. Each subpart of the division of a cm is 1mm.

> 1 mm is 0.1 cm.
> 2 mm is 0.2 cm and so on .
> 2.3 cm will mean 2 cm and 3 mm.



How many millimetres make one centimetre? Since 1cm = 10 mm, how will we write 2 cm? 3mm? What do we mean by 7.7 cm?

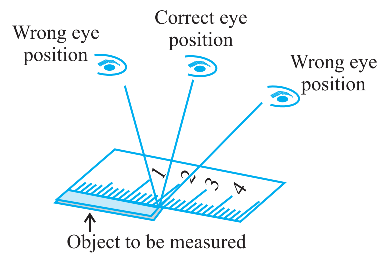
Place the zero mark of the ruler at A. Read the mark against B. This gives the length of $\overline{AB}$. Suppose the length is 5.8 cm, we may write,

Length AB = 5.8 cm or more simply as AB = 5.8 cm.

There is room for errors even in this procedure. The thickness of the ruler may cause difficulties in reading off the marks on it.

Think, discuss and write

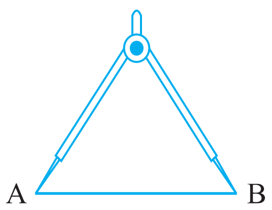
1. What other errors and difficulties might we face?
2. What kind of errors can occur if viewing the mark on the ruler is not proper? How can one avoid it?

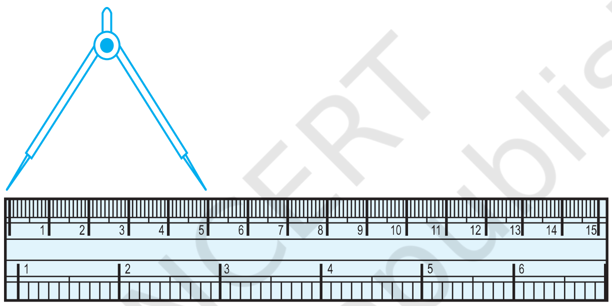


Positioning error

To get correct measure, the eye should be correctly positioned, just vertically above the mark. Otherwise errors can happen due to angular viewing.

Can we avoid this problem? Is there a better way?
Let us use the divider to measure length.





Open the divider. Place the end point of one of its arms at A and the end point of the second arm at B. Taking care that opening of the divider is not disturbed, lift the divider and place it on the ruler. Ensure that one end point is at the zero mark of the ruler. Now read the mark against the other end point.

Try These

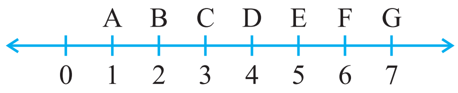
1. Take any post card. Use the above technique to measure its two adjacent sides.
2. Select any three objects having a flat top. Measure all sides of the top using a divider and a ruler.

EXERCISE 5.1

1. What is the disadvantage in comparing line segments by mere observation?
2. Why is it better to use a divider than a ruler, while measuring the length of a line segment?
3. Draw any line segment, say $\overline{AB}$. Take any point C lying in between A and B. Measure the lengths of AB, BC and AC. Is AB = AC + CB?

   [Note : If A,B,C are any three points on a line such that AC + CB = AB, then we can be sure that C lies between A and B.]

4. If A,B,C are three points on a line such that AB = 5 cm, BC = 3 cm and AC = 8 cm, which one of them lies between the other two?
5. Verify, whether D is the mid point of $\overline{AG}$.



6. If B is the mid point of $\overline{AC}$ and C is the mid point of $\overline{BD}$, where A,B,C,D lie on a straight line, say why AB = CD?
7. Draw five triangles and measure their sides. Check in each case, if the sum of the lengths of any two sides is always less than the third side.

5.3 Angles – 'Right' and 'Straight'

You have heard of directions in Geography. We know that China is to the north of India, Sri Lanka is to the south. We also know that Sun rises in the east and sets in the west. There are four main directions. They are North (N), South (S), East (E) and West (W).

Do you know which direction is opposite to north?

Which direction is opposite to west?

Just recollect what you know already. We now use this knowledge to learn a few properties about angles.

Stand facing north.

Do This

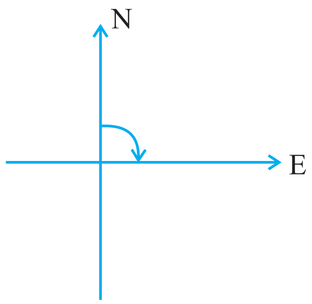
Turn clockwise to east.

We say, you have turned through a **right angle.**

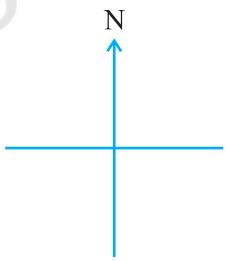
Follow this by a 'right-angle-turn', clockwise.

You now face south.

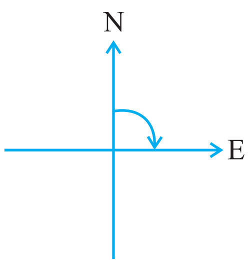
If you turn by a right angle in the anti-clockwise direction, which direction will you face? It is east again! (Why?)



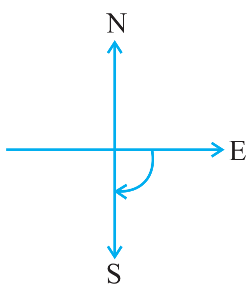
Study the following positions :



You stand facing north

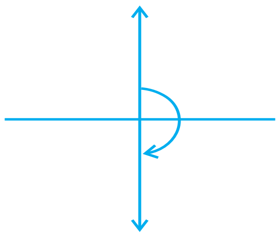


By a 'right-angle-turn' clockwise, you now face east



By another 'right-angle-turn' you finally face south.

From facing north to facing south, you have turned by two right angles. Is not this the same as a single turn by two right angles?



The turn from north to east is by a right angle.

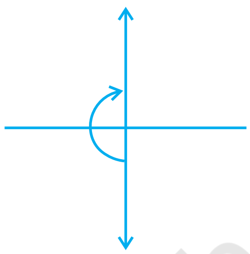
The turn from north to south is by two right angles; it is called a **straight angle**. (NS is a straight line!)

Stand facing south.

Turn by a straight angle.

Which direction do you face now?

You face north!



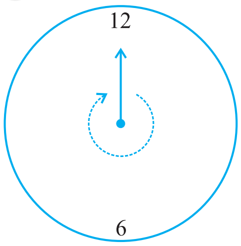
To turn from north to south, you took a straight angle turn, again to turn from south to north, you took another straight angle turn in the same direction. Thus, turning by two straight angles you reach your original position.

Think, discuss and write

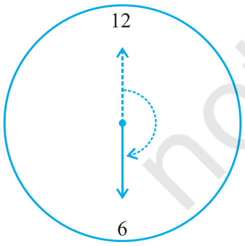
By how many right angles should you turn in the same direction to reach your original position?

Turning by two straight angles (or four right angles) in the same direction makes a full turn. This one complete turn is called one revolution. The angle for one revolution is a **complete angle**.

We can see such revolutions on clock-faces. When the hand of a clock moves from one position to another, it turns through an **angle.**



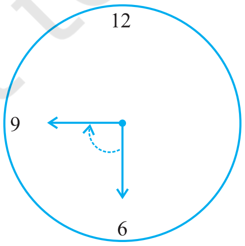
Suppose the hand of a clock starts at 12 and goes round until it reaches at 12 again. Has it not made one revolution? So, how many right angles has it moved? Consider these examples :



From 12 to 6

$\frac{1}{2}$ of a revolution.

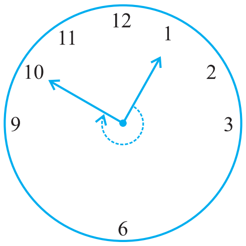
or 2 right angles.



From 6 to 9

$\frac{1}{4}$ of a revolution

or 1 right angle.



From 1 to 10

$\frac{3}{4}$ of a revolution

or 3 right angles.

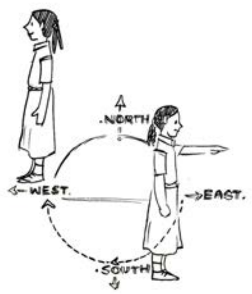
Try These

1. What is the angle name for half a revolution?
2. What is the angle name for one-fourth revolution?
3. Draw five other situations of one-fourth, half and three-fourth revolution on a clock.

Note that there is no special name for three-fourth of a revolution.

EXERCISE 5.2

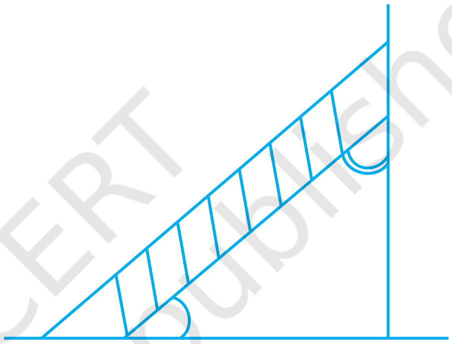
1. What fraction of a clockwise revolution does the hour hand of a clock turn through, when it goes from
   (a) 3 to 9 (b) 4 to 7 (c) 7 to 10
   (d) 12 to 9 (e) 1 to 10 (f) 6 to 3
2. Where will the hand of a clock stop if it
   (a) starts at 12 and makes $\frac{1}{2}$ of a revolution, clockwise?
   (b) starts at 2 and makes $\frac{1}{2}$ of a revolution, clockwise?
   (c) starts at 5 and makes $\frac{1}{4}$ of a revolution, clockwise?
   (d) starts at 5 and makes $\frac{3}{4}$ of a revolution, clockwise?
3. Which direction will you face if you start facing
   (a) east and make $\frac{1}{2}$ of a revolution clockwise?
   (b) east and make $1\frac{1}{2}$ of a revolution clockwise?
   (c) west and make $\frac{3}{4}$ of a revolution anti-clockwise?
   (d) south and make one full revolution?
   (Should we specify clockwise or anti-clockwise for this last question? Why not?)



4. What part of a revolution have you turned through if you stand facing
   (a) east and turn clockwise to face north?
   (b) south and turn clockwise to face east?
   (c) west and turn clockwise to face east?
5. Find the number of right angles turned through by the hour hand of a clock when it goes from
   (a) 3 to 6 (b) 2 to 8 (c) 5 to 11
   (d) 10 to 1 (e) 12 to 9 (f) 12 to 6

6. How many right angles do you make if you start facing
   (a) south and turn clockwise to west?
   (b) north and turn anti-clockwise to east?
   (c) west and turn to west?
   (d) south and turn to north?
7. Where will the hour hand of a clock stop if it starts
   (a) from 6 and turns through 1 right angle?
   (b) from 8 and turns through 2 right angles?
   (c) from 10 and turns through 3 right angles?
   (d) from 7 and turns through 2 straight angles?

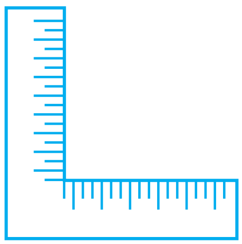
5.4 Angles – 'Acute', 'Obtuse' and 'Reflex'



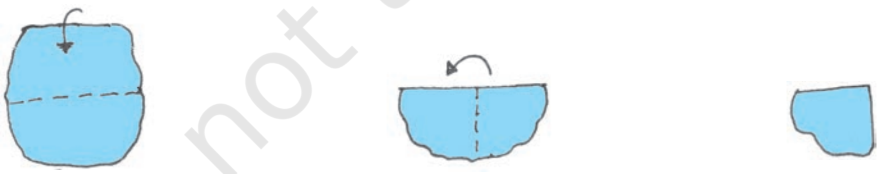
We saw what we mean by a right angle and a straight angle. However, not all the angles we come across are one of these two kinds. The angle made by a ladder with the wall (or with the floor) is neither a right angle nor a straight angle.

Think, discuss and write

Are there angles smaller than a right angle?
Are there angles greater than a right angle?
Have you seen a carpenter's square? It looks like the letter "L" of English alphabet. He uses it to check right angles. Let us also make a similar 'tester' for a right angle.



Do This

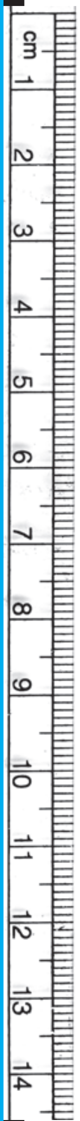


Step 1
Take a piece of paper

Step 2
Fold it somewhere in the middle

Step 3
Fold again the straight edge. Your tester is ready

Observe your improvised 'right-angle-tester'. [Shall we call it RA tester?] Does one edge end up straight on the other?

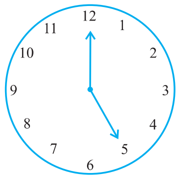


Suppose any shape with corners is given. You can use your RA tester to test the angle at the corners.

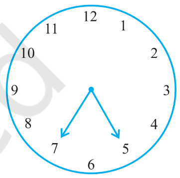
Do the edges match with the angles of a paper? If yes, it indicates a right angle.

Try These

1. The hour hand of a clock moves from 12 to 5. Is the revolution of the hour hand more than 1 right angle?



2. What does the angle made by the hour hand of the clock look like when it moves from 5 to 7. Is the angle moved more than 1 right angle?

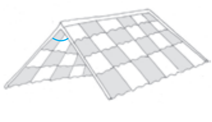


3. Draw the following and check the angle with your RA tester.
   (a) going from 12 to 2 (b) from 6 to 7
   (c) from 4 to 8 (d) from 2 to 5
4. Take five different shapes with corners. Name the corners. Examine them with your tester and tabulate your results for each case :

Corner	**Smaller than** 	**Larger than** 
A	....	....
B	....	....
C	....	....
⋮		

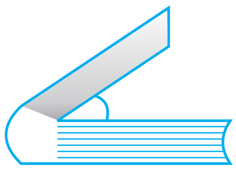
Other names

- An angle smaller than a right angle is called an **acute angle**. These are acute angles.



Roof top

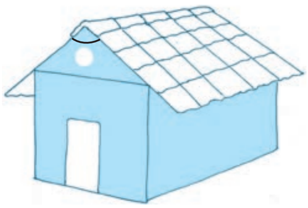
Sea-saw



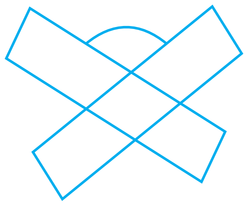
Opening book

Do you see that each one of them is less than one-fourth of a revolution? Examine them with your RA tester.

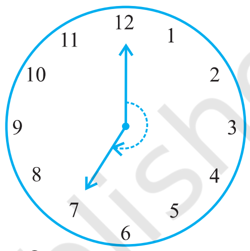
- If an angle is larger than a right angle, but less than a straight angle, it is called an **obtuse angle**. These are obtuse angles.



House



Book reading desk



Do you see that each one of them is greater than one-fourth of a revolution but less than half a revolution? Your RA tester may help to examine.

Identify the obtuse angles in the previous examples too.

- A reflex angle is larger than a straight angle.
  It looks like this. (See the angle mark)
  Were there any reflex angles in the shapes you made earlier?
  How would you check for them?

Try These

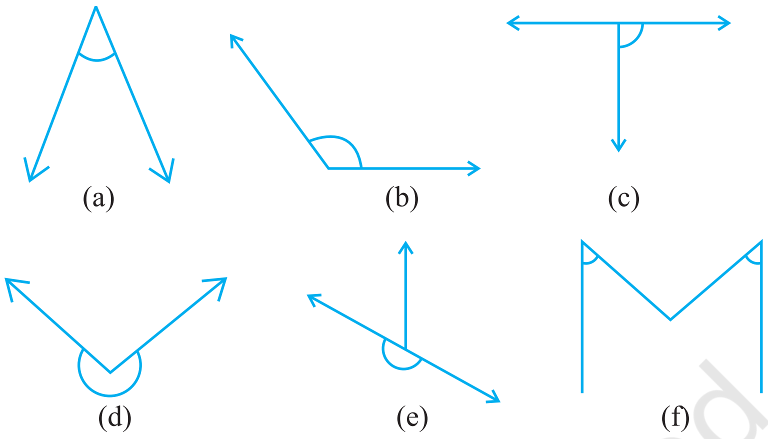
1. Look around you and identify edges meeting at corners to produce angles. List ten such situations.
2. List ten situations where the angles made are acute.
3. List ten situations where the angles made are right angles.
4. Find five situations where obtuse angles are made.
5. List five other situations where reflex angles may be seen.

EXERCISE 5.3

1. Match the following :

(i)	Straight angle	(a)	Less than one-fourth of a revolution
(ii)	Right angle	(b)	More than half a revolution
(iii)	Acute angle	(c)	Half of a revolution
(iv)	Obtuse angle	(d)	One-fourth of a revolution
(v)	Reflex angle	(e)	Between $\frac{1}{4}$ and $\frac{1}{2}$ of a revolution
		(f)	One complete revolution

2. Classify each one of the following angles as right, straight, acute, obtuse or reflex :



5.5 Measuring Angles

The improvised ‘Right-angle tester’ we made is helpful to compare angles with a right angle. We were able to classify the angles as acute, obtuse or reflex.

But this does not give a precise comparison. It cannot find which one among the two obtuse angles is greater. So in order to be more precise in comparison, we need to ‘measure’ the angles. We can do it with a ‘protractor’.

The measure of angle

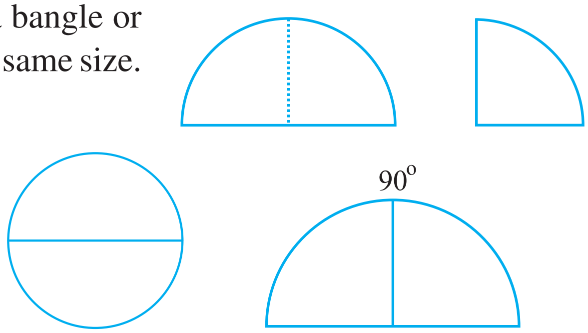
We call our measure, ‘degree measure’. One complete revolution is divided into 360 equal parts. Each part is a **degree**. We write 360° to say ‘three hundred sixty degrees’.

Think, discuss and write

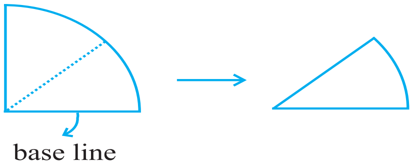
How many degrees are there in half a revolution? In one right angle? In one straight angle?
How many right angles make 180°? 360°?

Do This

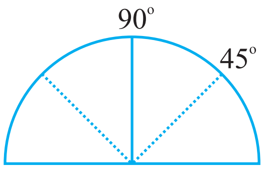
1. Cut out a circular shape using a bangle or take a circular sheet of about the same size.
2. Fold it twice to get a shape as shown. This is called a quadrant.
3. Open it out. You will find a semi-circle with a fold in the middle. Mark 90° on the fold.



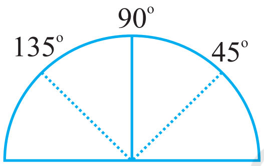
4. Fold the semicircle to reach the quadrant. Now fold the quadrant once more as shown. The angle is half of 90° i.e. 45°.



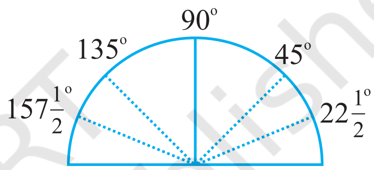
5. Open it out now. Two folds appear on each side. What is the angle upto the first new line? Write 45° on the first fold to the left of the base line.



6. The fold on the other side would be 90° + 45° = 135°



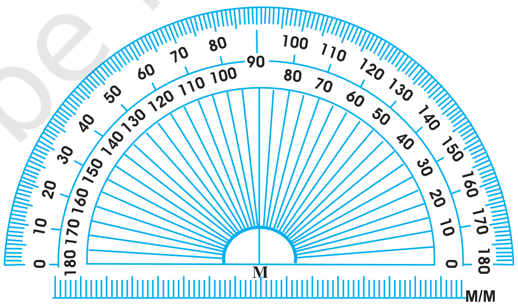
7. Fold the paper again upto 45° (half of the quadrant). Now make half of this. The first fold to the left of the base line now is half of 45° i.e. $22\frac{1}{2}^{\circ}$. The angle on the left of 135° would be $157\frac{1}{2}^{\circ}$.



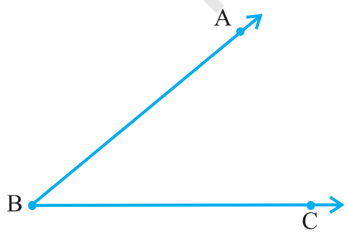
You have got a ready device to measure angles. This is an approximate protractor.

The Protractor

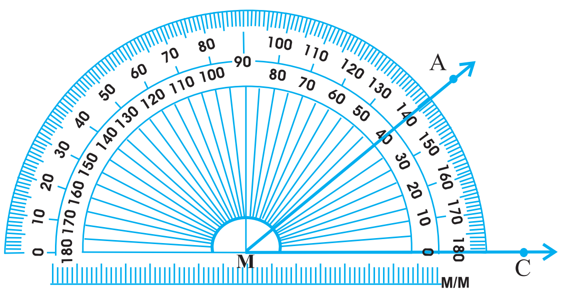
You can find a readymade protractor in your 'instrument box'. The curved edge is divided into 180 equal parts. Each part is equal to a 'degree'. The markings start from 0° on the right side and ends with 180° on the left side, and vice-versa.



Suppose you want to measure an angle ABC.



Given ∠ABC



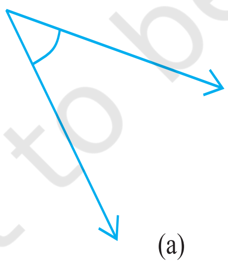
Measuring ∠ABC

1. Place the protractor so that the mid point (M in the figure) of its straight edge lies on the vertex B of the angle.
2. Adjust the protractor so that $\overline{BC}$ is along the straight-edge of the protractor.
3. There are two ‘scales’ on the protractor : read that scale which has the 0° mark coinciding with the straight-edge (i.e. with ray $\overrightarrow{BC}$).
4. The mark shown by $\overline{BA}$ on the curved edge gives the degree measure of the angle.

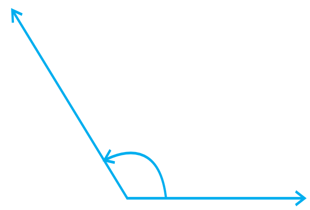
   We write m $\angle ABC = 40°$, or simply $\angle ABC = 40°$.

EXERCISE 5.4

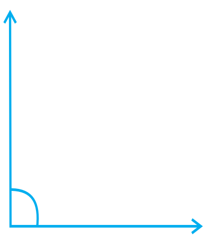
1. What is the measure of (i) a right angle? (ii) a straight angle?
2. Say True or False :
   (a) The measure of an acute angle < 90°.
   (b) The measure of an obtuse angle < 90°.
   (c) The measure of a reflex angle > 180°.
   (d) The measure of one complete revolution = 360°.
   (e) If $m\angle A = 53°$ and $m\angle B = 35°$, then $m\angle A > m\angle B$.
3. Write down the measures of
   (a) some acute angles. (b) some obtuse angles.
   (give at least two examples of each).
4. Measure the angles given below using the Protractor and write down the measure.



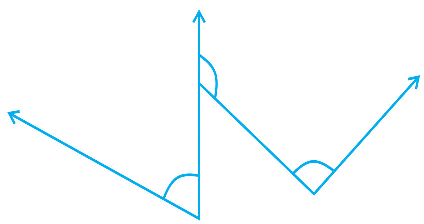
(a)



(b)



(c)

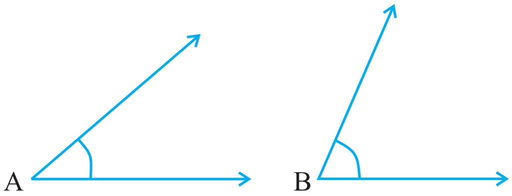


(d)

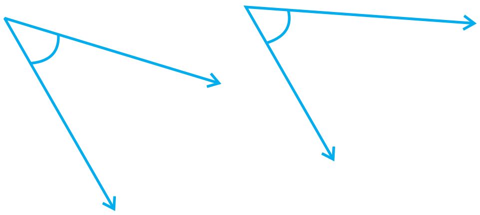
5. Which angle has a large measure? First estimate and then measure.

   Measure of Angle A =

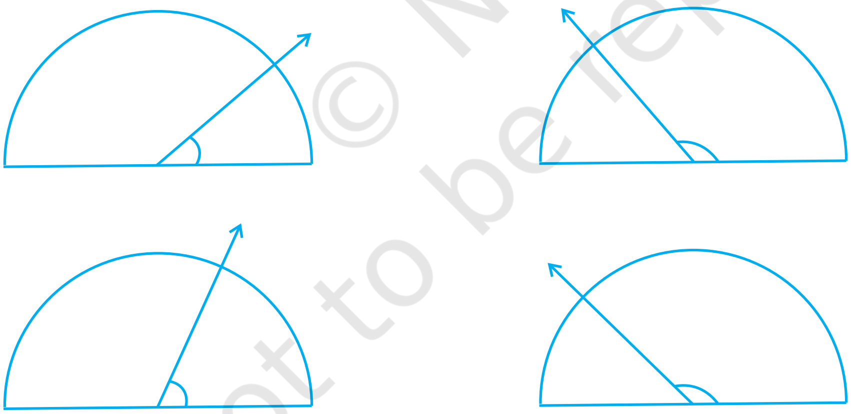
   Measure of Angle B =



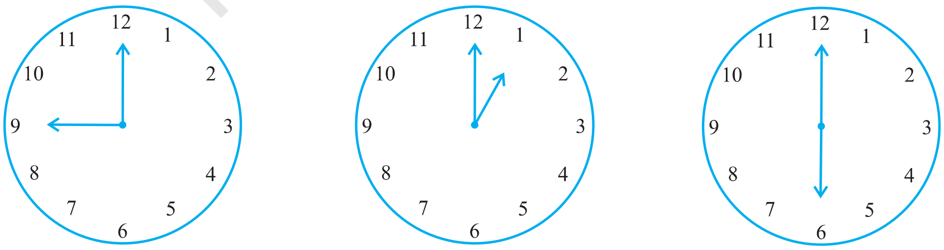
6. From these two angles which has larger measure? Estimate and then confirm by measuring them.



7. Fill in the blanks with acute, obtuse, right or straight :
   (a) An angle whose measure is less than that of a right angle is______.
   (b) An angle whose measure is greater than that of a right angle is ______.
   (c) An angle whose measure is the sum of the measures of two right angles is _____.
   (d) When the sum of the measures of two angles is that of a right angle, then each one of them is ______.
   (e) When the sum of the measures of two angles is that of a straight angle and if one of them is acute then the other should be _______.
8. Find the measure of the angle shown in each figure. (First estimate with your eyes and then find the actual measure with a protractor).



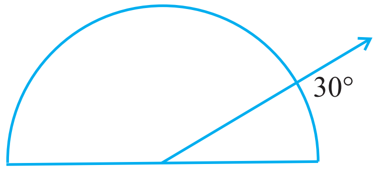
9. Find the angle measure between the hands of the clock in each figure :



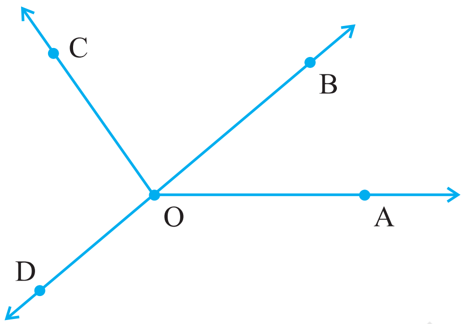
9.00 a.m. 1.00 p.m. 6.00 p.m.

10. **Investigate**

In the given figure, the angle measures 30°. Look at the same figure through a magnifying glass. Does the angle becomes larger? Does the size of the angle change?



11. Measure and classify each angle :



Angle	Measure	Type
∠AOB		
∠AOC		
∠BOC		
∠DOC		
∠DOA		
∠DOB		

5.6 Perpendicular Lines

When two lines intersect and the angle between them is a right angle, then the lines are said to be **perpendicular**. If a line AB is perpendicular to CD, we write $AB \perp CD$.

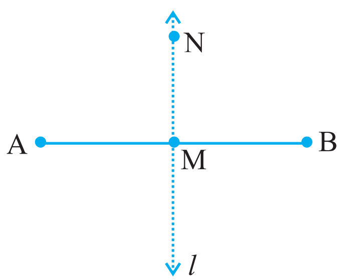
Think, discuss and write

If $AB \perp CD$, then should we say that $CD \perp AB$ also?

Perpendiculars around us!

You can give plenty of examples from things around you for perpendicular lines (or line segments). The English alphabet T is one. Is there any other alphabet which illustrates perpendicularity?

Consider the edges of a post card. Are the edges perpendicular?



Let $\overline{AB}$ be a line segment. Mark its mid point as M. Let MN be a line perpendicular to $\overline{AB}$ through M.

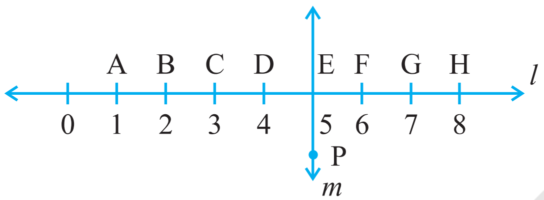
Does MN divide $\overline{AB}$ into two equal parts?

MN bisects $\overline{AB}$ (that is, divides $\overline{AB}$ into two equal parts) and is also perpendicular to $\overline{AB}$.

So we say MN is the **perpendicular bisector** of $\overline{AB}$. You will learn to construct it later.

EXERCISE 5.5

1. Which of the following are models for perpendicular lines :
   (a) The adjacent edges of a table top.
   (b) The lines of a railway track.
   (c) The line segments forming the letter 'L'.
   (d) The letter V.
2. Let $\overline{PQ}$ be the perpendicular to the line segment $\overline{XY}$. Let $\overline{PQ}$ and $\overline{XY}$ intersect in the point A. What is the measure of $\angle PAY$?
3. There are two set-squares in your box. What are the measures of the angles that are formed at their corners? Do they have any angle measure that is common?
4. Study the diagram. The line *l* is perpendicular to line *m*
   (a) Is CE = EG?



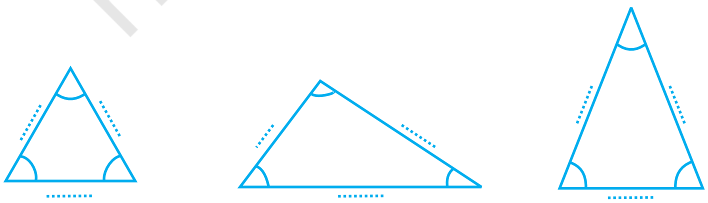
   (b) Does PE bisect CG?
   (c) Identify any two line segments for which PE is the perpendicular bisector.
   (d) Are these true?
      (i) AC > FG
      (ii) CD = GH
      (iii) BC < EH.

5.7 Classification of Triangles

Do you remember a polygon with the least number of sides? That is a triangle. Let us see the different types of triangle we can get.

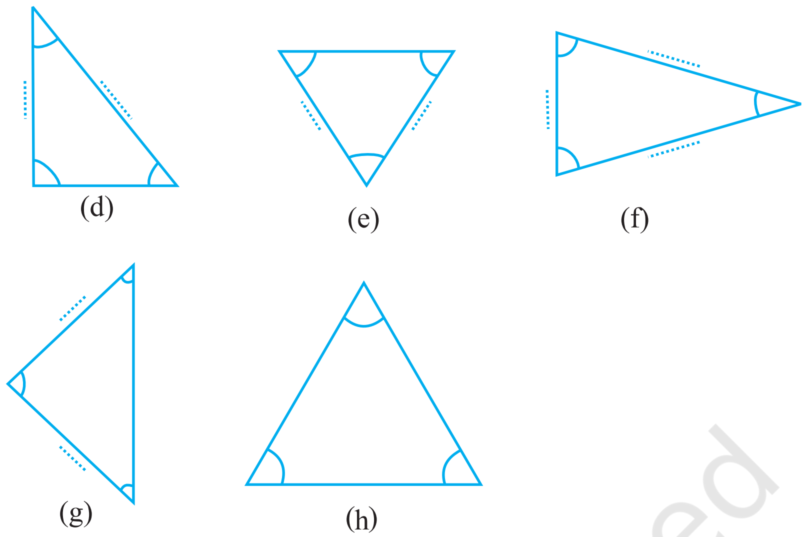
Do This

Using a protractor and a ruler find the measures of the sides and angles of the given triangles. Fill the measures in the given table.



(a) (b) (c)

Rationalised 2023-24



The measure of the angles of the triangle	What can you say about the angles?	Measures of the sides
(a) ...60⁰..., ...60⁰.., ...60⁰.....,	All angles are equal	
(b) ...., ..., ...,	.... angles ....,	
(c) ...., ..., ...,	.... angles ....,	
(d) ...., ..., ...,	.... angles ....,	
(e) ...., ..., ...,	.... angles ....,	
(f) ...., ..., ...,	.... angles ....,	
(g) ...., ..., ...,	.... angles ....,	
(h) ...., ..., ...,	.... angles ....,	

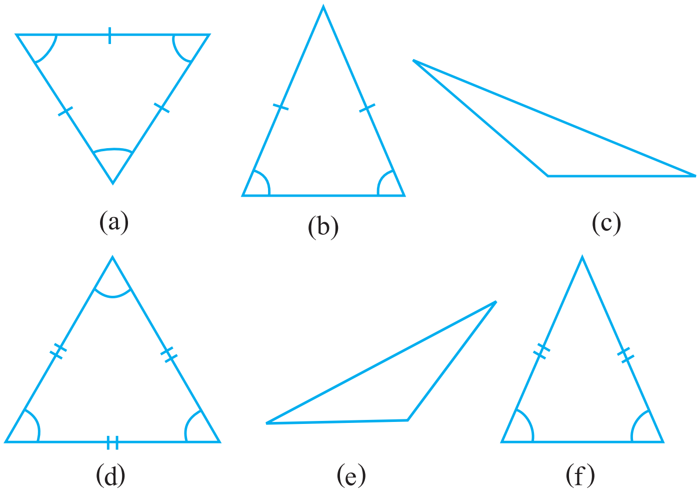
Observe the angles and the triangles as well as the measures of the sides carefully. Is there anything special about them?

What do you find?

- Triangles in which all the angles are equal.
  If all the angles in a triangle are equal, then its sides are also ....
- Triangles in which all the three sides are equal.
  If all the sides in a triangle are equal, then its angles are............. .
- Triangle which have two equal angles and two equal sides.
  If two sides of a triangle are equal, it has .... equal angles.
  and if two angles of a triangle are equal, it has .... equal sides.
- Triangles in which no two sides are equal.
  If none of the angles of a triangle are equal then none of the sides are equal.
  If the three sides of a triangle are unequal then, the three angles are also............. .

Take some more triangles and verify these. For this we will again have to measure all the sides and angles of the triangles.

The triangles have been divided into categories and given special names. Let us see what they are.



Naming triangles based on sides

A triangle having all three unequal sides is called a **Scalene Triangle** [(c), (e)].

A triangle having two equal sides is called an **Isosceles Triangle** [(b), (f)].

A triangle having three equal sides is called an **Equilateral Triangle** [(a), (d)].

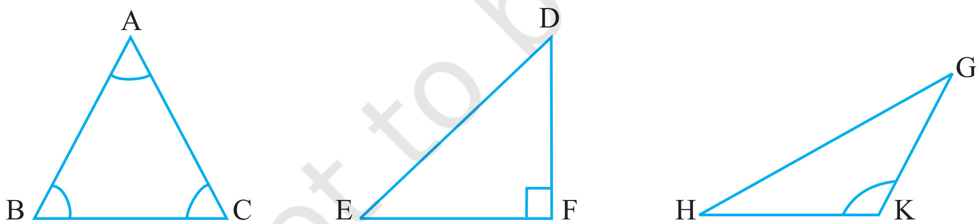
Classify all the triangles whose sides you measured earlier, using these definitions.

Naming triangles based on angles

If each angle is less than 90°, then the triangle is called an **acute angled triangle**.

If any one angle is a right angle then the triangle is called a **right angled triangle**.

If any one angle is greater than 90°, then the triangle is called an **obtuse angled triangle**.



Acute Angled Triangle   Right Angled Triangle   Obtuse Angled Triangle

Name the triangles whose angles were measured earlier according to these three categories. How many were right angled triangles?

Do This

Try to draw rough sketches of

(a) a scalene acute angled triangle.

(b) an obtuse angled isosceles triangle.

(c) a right angled isosceles triangle.

(d) a scalene right angled triangle.

Do you think it is possible to sketch

(a) an obtuse angled equilateral triangle ?

(b) a right angled equilateral triangle ?

(c) a triangle with two right angles?

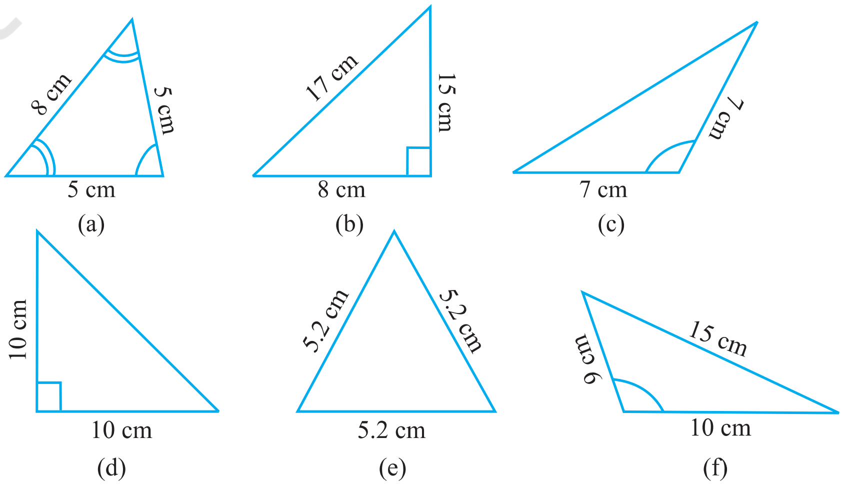
Think, discuss and write your conclusions.

EXERCISE 5.6

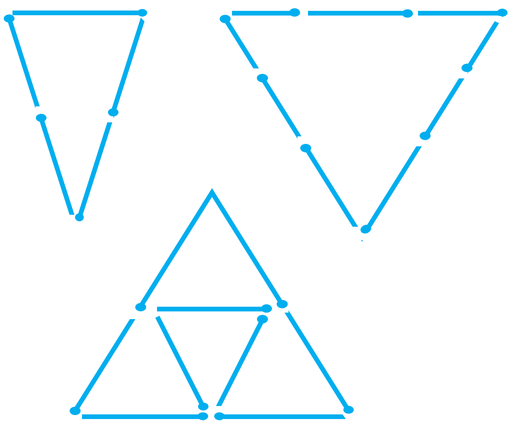
1. Name the types of following triangles :
   (a) Triangle with lengths of sides 7 cm, 8 cm and 9 cm.
   (b) ΔABC with AB = 8.7 cm, AC = 7 cm and BC = 6 cm.
   (c) ΔPQR such that PQ = QR = PR = 5 cm.
   (d) ΔDEF with $m\angle D = 90°$
   (e) ΔXYZ with $m\angle Y = 90°$ and XY = YZ.
   (f) ΔLMN with $m\angle L = 30°$, $m\angle M = 70°$ and $m\angle N = 80°$.
2. Match the following :

Measures of Triangle	Type of Triangle
(i) 3 sides of equal length	(a) Scalene
(ii) 2 sides of equal length	(b) Isosceles right angled
(iii) All sides are of different length	(c) Obtuse angled
(iv) 3 acute angles	(d) Right angled
(v) 1 right angle	(e) Equilateral
(vi) 1 obtuse angle	(f) Acute angled
(vii) 1 right angle with two sides of equal length	(g) Isosceles

3. Name each of the following triangles in two different ways: (you may judge the nature of the angle by observation)



4. Try to construct triangles using match sticks. Some are shown here.
   Can you make a triangle with
   (a) 3 matchsticks?
   (b) 4 matchsticks?
   (c) 5 matchsticks?
   (d) 6 matchsticks?
   (Remember you have to use all the available matchsticks in each case)
   Name the type of triangle in each case.
   If you cannot make a triangle, think of reasons for it.

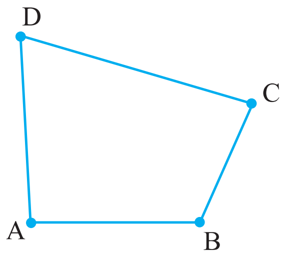


5.8 Quadrilaterals

A quadrilateral, if you remember, is a polygon which has four sides.

Do This

1. Place a pair of unequal sticks such that they have their end points joined at one end. Now place another such pair meeting the free ends of the first pair.
   What is the figure enclosed?
   It is a quadrilateral, like the one you see here.
   The sides of the quadrilateral are $\overline{AB}$, $\overline{BC}$, ___, ___.
   There are 4 angles for this quadrilateral.
   They are given by $\angle BAD$, $\angle ADC$, $\angle DCB$ and _____.
   BD is one diagonal. What is the other?
   Measure the length of the sides and the diagonals.
   Measure all the angles also.



2. Using four unequal sticks, as you did in the above activity, see if you can form a quadrilateral such that
   (a) all the four angles are acute.
   (b) one of the angles is obtuse.
   (c) one of the angles is right angled.
   (d) two of the angles are obtuse.
   (e) two of the angles are right angled.
   (f) the diagonals are perpendicular to one another.

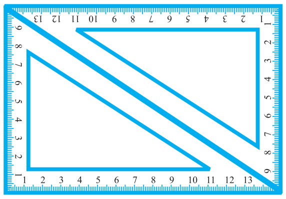
Do This

You have two set-squares in your instrument box. One is 30° – 60° – 90° set-square, the other is 45°– 45°– 90° set square.

You and your friend can jointly do this.

(a) Both of you will have a pair of 30°– 60°– 90° set-squares. Place them as shown in the figure.

Can you name the quadrilateral described?



What is the measure of each of its angles?

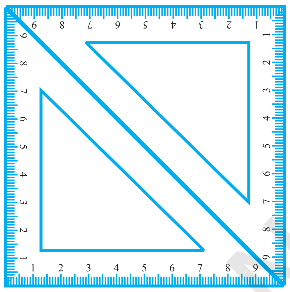
This quadrilateral is a **rectangle.**

One more obvious property of the rectangle you can see is that opposite sides are of equal length.

What other properties can you find?

(b) If you use a pair of 45°– 45°–90° set-squares, you get another quadrilateral this time.

It is a **square**.

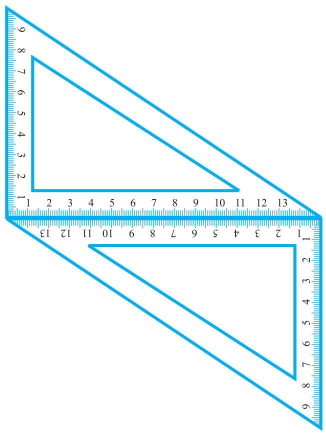


Are you able to see that all the sides are of equal length? What can you say about the angles and the diagonals? Try to find a few more properties of the square.

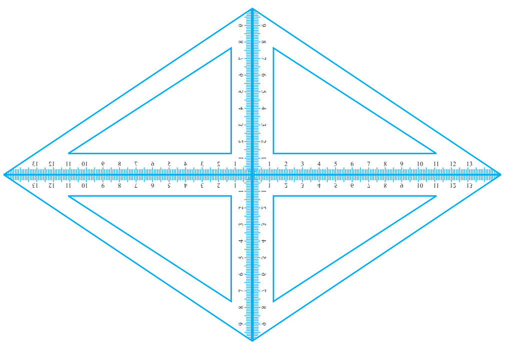
(c) If you place the pair of 30° – 60° – 90° set-squares in a different position, you get a **parallelogram**. Do you notice that the opposite sides are parallel?

Are the opposite sides equal?

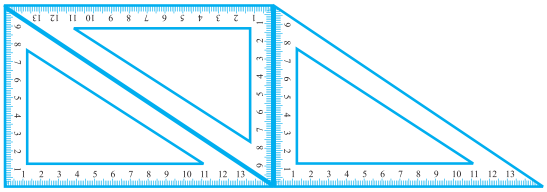
Are the diagonals equal?



(d) If you use four 30° – 60° – 90° set-squares you get a **rhombus**.



(e) If you use several set-squares you can build a shape like the one given here.



Here is a quadrilateral in which a pair of two opposite sides is parallel.

It is a **trapezium.**

Here is an outline-summary of your possible findings. Complete it.

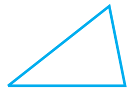
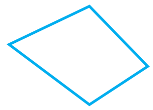
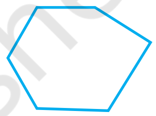
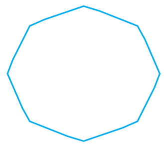
Quadrilateral	Opposite sides		All sides	Opposite Angles	Diagonals	
	Parallel	Equal	Equal	Equal	Equal	Perpen-dicular
Parallelogram	Yes	Yes	No	Yes	No	No
Rectangle			No			
Square						Yes
Rhombus				Yes		
Trapezium		No				

EXERCISE 5.7

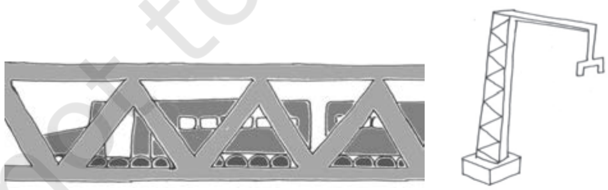
1. Say True or False :
   (a) Each angle of a rectangle is a right angle.
   (b) The opposite sides of a rectangle are equal in length.
   (c) The diagonals of a square are perpendicular to one another.
   (d) All the sides of a rhombus are of equal length.
   (e) All the sides of a parallelogram are of equal length.
   (f) The opposite sides of a trapezium are parallel.
2. Give reasons for the following :
   (a) A square can be thought of as a special rectangle.
   (b) A rectangle can be thought of as a special parallelogram.
   (c) A square can be thought of as a special rhombus.
   (d) Squares, rectangles, parallelograms are all quadrilaterals.
   (e) Square is also a parallelogram.
3. A figure is said to be regular if its sides are equal in length and angles are equal in measure. Can you identify the regular quadrilateral?

5.9 Polygons

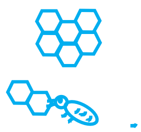
So far you studied polygons of 3 or 4 sides (known as triangles and quardrilaterals respectively). We now try to extend the idea of polygon to figures with more number of sides. We may classify polygons according to the number of their sides.

Number of sides	Name	Illustration
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
8	Octagon	

You can find many of these shapes in everyday life. Windows, doors, walls, almirahs, blackboards, notebooks are all usually rectanglular in shape. Floor tiles are rectangles. The sturdy nature of a triangle makes it the most useful shape in engineering constructions.



The triangle finds application in constructions.

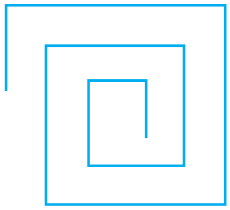


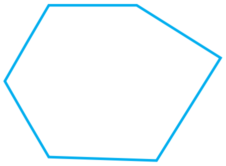
A bee knows the usefulness of a hexagonal shape in building its house .

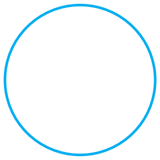
Look around and see where you can find all these shapes.

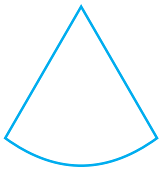
EXERCISE 5.8

1. Examine whether the following are polygons. If any one among them is not, say why?

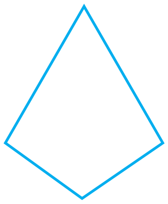

(a)

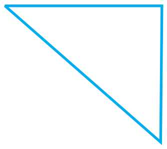

(b)

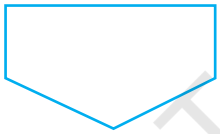

(c)

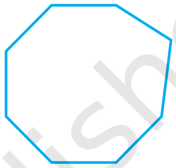

(d)

2. Name each polygon.


(a)


(b)


(c)


(d)

   Make two more examples of each of these.

3. Draw a rough sketch of a regular hexagon. Connecting any three of its vertices, draw a triangle. Identify the type of the triangle you have drawn.
4. Draw a rough sketch of a regular octagon. (Use squared paper if you wish). Draw a rectangle by joining exactly four of the vertices of the octagon.
5. A diagonal is a line segment that joins any two vertices of the polygon and is not a side of the polygon. Draw a rough sketch of a pentagon and draw its diagonals.

What have we discussed?

1. The distance between the end points of a line segment is its *length*.
2. A graduated *ruler* and the *divider* are useful to compare lengths of line segments.
3. When a hand of a clock moves from one position to another position we have an example for an *angle*.

   One full turn of the hand is 1 *revolution*.

   A *right angle* is ¼ revolution and a *straight angle* is ½ a revolution .

   We use a *protractor* to measure the size of an angle in degrees.

   The measure of a right angle is 90° and hence that of a straight angle is 180°.

   An angle is *acute* if its measure is smaller than that of a right angle and is *obtuse* if its measure is greater than that of a right angle and less than a straight angle.

   A *reflex angle* is larger than a straight angle.

4. Two intersecting lines are *perpendicular* if the angle between them is 90°.
5. The *perpendicular bisector* of a line segment is a perpendicular to the line segment that divides it into two equal parts.
6. Triangles can be classified as follows based on their angles:

Nature of angles in the triangle	*Name*
Each angle is acute	Acute angled triangle
One angle is a right angle	Right angled triangle
One angle is obtuse	Obtuse angled triangle

7. Triangles can be classified as follows based on the lengths of their sides:

Nature of sides in the triangle	*Name*
All the three sides are of unequal length	Scalene triangle
Any two of the sides are of equal length	Isosceles triangle
All the three sides are of equal length	Equilateral triangle

8. Polygons are named based on their sides.

Number of sides	*Name of the Polygon*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
8	Octagon

9. Quadrilaterals are further classified with reference to their properties.

Properties	*Name of the Quadrilateral*
One pair of parallel sides	Trapezium
Two pairs of parallel sides	Parallelogram
Parallelogram with 4 right angles	Rectangle
Parallelogram with 4 sides of equal length	Rhombus
A rhombus with 4 right angles	Square

Integers

Chapter 6

0650CH06

6.1 Introduction

Sunita's mother has 8 bananas. Sunita has to go for a picnic with her friends. She wants to carry 10 bananas with her. Can her mother give 10 bananas to her? She does not have enough, so she borrows 2 bananas from her neighbour to be returned later. After giving 10 bananas to Sunita, how many bananas are left with her mother? Can we say that she has zero bananas? She has no bananas with her, but has to return two to her neighbour. So when she gets some more bananas, say 6, she will return 2 and be left with 4 only.

Ronald goes to the market to purchase a pen. He has only ₹ 12 with him but the pen costs ₹ 15. The shopkeeper writes ₹ 3 as due amount from him. He writes ₹ 3 in his diary to remember Ronald's debit. But how would he remember whether ₹ 3 has to be given or has to be taken from Ronald? Can he express this debit by some colour or sign?

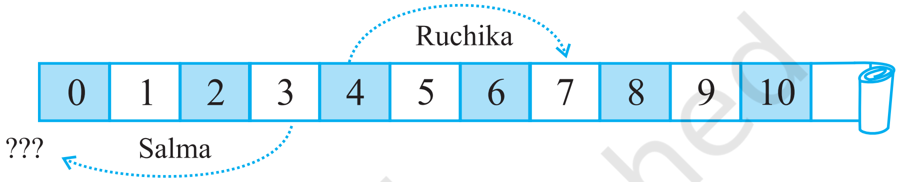
Ruchika and Salma are playing a game using a number strip which is marked from 0 to 25 at equal intervals.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

To begin with, both of them placed a coloured token at the zero mark. Two coloured dice are placed in a bag and are taken out by them one by one. If the die is red in colour, the token is moved forward as per the number shown on throwing this die. If it is blue, the token is moved backward as per the number shown when this die is thrown. The dice are put back into the bag after each move so that both of them have equal chance of getting either die. The one who reaches the 25th mark first is the winner. They play the game. Ruchika gets the red die and gets four on the die after throwing it. She, thus, moves the token to mark four on the strip. Salma also happens to take out the red die and wins 3 points and, thus, moves her token to number 3.

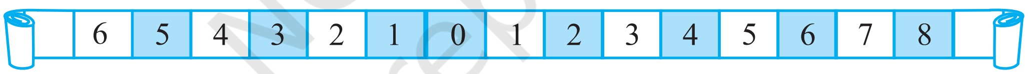
In the second attempt, Ruchika secures three points with the red die and Salma gets 4 points but with the blue die. Where do you think both of them should place their token after the second attempt?

Ruchika moves forward and reaches 4 + 3 i.e. the 7th mark.



Whereas Salma placed her token at zero position. But Ruchika objected saying she should be behind zero. Salma agreed. But there is nothing behind zero. What can they do?

Salma and Ruchika then extended the strip on the other side. They used a blue strip on the other side.



Now, Salma suggested that she is one mark behind zero, so it can be marked as blue one. If the token is at blue one, then the position behind blue one is blue two. Similarly, blue three is behind blue two. In this way they decided to move backward. Another day while playing they could not find blue paper, so Ruchika said, let us use a sign on the other side as we are moving in opposite direction. So you see we need to use a sign going for numbers less than zero. The sign that is used is the placement of a minus sign attached to the number. This indicates that numbers with a negative sign are less than zero. These are called ***negative numbers*.**

Do This

(Who is where?)

Suppose David and Mohan have started walking from zero position in opposite directions. Let the steps to the right of zero be represented by '+' sign and to the left of zero represented by '–' sign. If Mohan moves 5 steps to the right of zero it can be represented as +5 and if David moves 5 steps to the left of zero it can be represented as – 5. Now represent the following positions with + or – sign :

(a) 8 steps to the left of zero.

(b) 7 steps to the right of zero.

(c) 11 steps to the right of zero.

(d) 6 steps to the left of zero.

Do This

(Who follows me?)

We have seen from the previous examples that a movement to the right is made if the number by which we have to move is positive. If a movement of only 1 is made we get the successor of the number.

Write the succeeding number of the following :

Number	Successor
10	
8	
– 5	
– 3	
0	

A movement to the left is made if the number by which the token has to move is negative.

If a movement of only 1 is made to the left, we get the predecessor of a number.

–8	–7	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5	6	7	8

Now write the preceding number of the following :

Number	Predecessor
10	
8	
5	
3	
0	

6.1.1 Tag me with a sign

We have seen that some numbers carry a minus sign. For example, if we want to show Ronald's due amount to the shopkeeper we would write it as – 3.

Following is an account of a shopkeeper which shows profit and loss from the sale of certain items. Since profit and loss are opposite situations and if profit is represented by '+' sign, loss can be represented by '–' sign.

Some of the situations where we may use these signs are :

Name of items	Profit	Loss	Representation with proper sign
Mustard oil	₹ 150		....
Rice		₹ 250	....
Black pepper	₹ 225		....
Wheat	₹ 200		....
Groundnut oil		₹ 330	....

The height of a place above sea level is denoted by a positive number. Height becomes lesser and lesser as we go lower and lower. Thus, below the surface of the sea level we can denote the height by a negative number.

Try These

Write the following numbers with appropriate signs :

(a) 100 m below sea level.
(b) 25°C above 0°C temperature.
(c) 15°C below 0°C temperature.
(d) any five numbers less than 0.

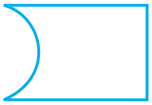
If earnings are represented by '+' sign, then the spendings may be shown by a '–' sign. Similarly, temperature above 0°C is denoted a '+' sign and temperature below 0°C is denoted by '–' sign.

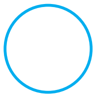
For example, the temperature of a place 10° below 0°C is written as –10°C.

6.2 Integers

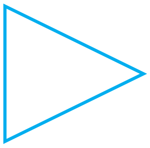
The first numbers to be discovered were natural numbers i.e. 1, 2, 3, 4,... If we include zero to the collection of natural numbers, we get a new collection of numbers known as whole numbers i.e. 0, 1, 2, 3, 4,... You have studied these numbers in the earlier chapter. Now we find that there are negative numbers too. If we put the whole numbers and the negative numbers together, the new collection of numbers will look like 0, 1, 2, 3, 4, 5,..., –1, – 2, – 3, –4, –5, ... and this collection of numbers is known as Integers. In this collection, 1, 2, 3, ... are said to be positive integers and – 1, – 2, – 3,.... are said to be negative integers.

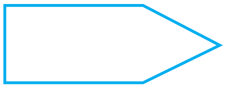
Let us understand this by the following figures. Let us suppose that the figures represent the collection of numbers written against them.


Natural numbers

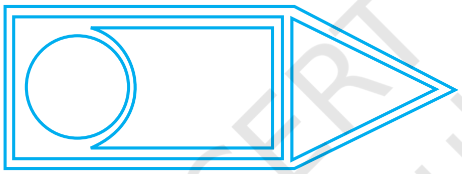

Zero


Whole numbers

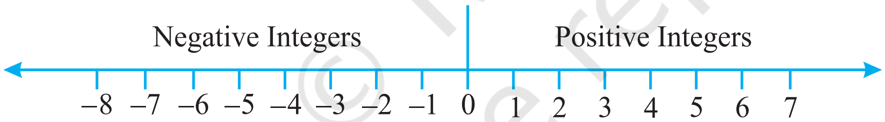

Negative numbers


Integers

Then the collection of integers can be understood by the following diagram in which all the earlier collections are included :

Integers


6.2.1 Representation of integers on a number line



Draw a line and mark some points at equal distance on it as shown in the figure. Mark a point as zero on it. Points to the right of zero are positive integers and are marked + 1, + 2, + 3, etc. or simply 1, 2, 3 etc. Points to the left of zero are negative integers and are marked – 1, – 2, – 3 etc.

In order to mark – 6 on this line, we move 6 points to the left of zero. (Fig 6.1)

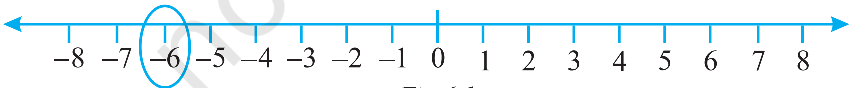


Fig 6.1

In order to mark + 2 on the number line, we move 2 points to the right of zero. (Fig 6.2)

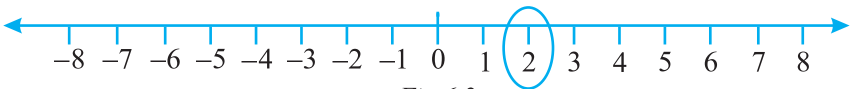


Fig 6.2

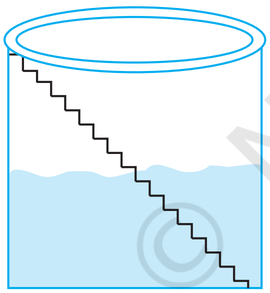
6.2.2 Ordering of integers

> **Try These**
>
> Mark –3, 7, –4, –8, –1 and – 3 on the number line.

Raman and Imran live in a village where there is a step well. There are in all 25 steps down to the bottom of the well.

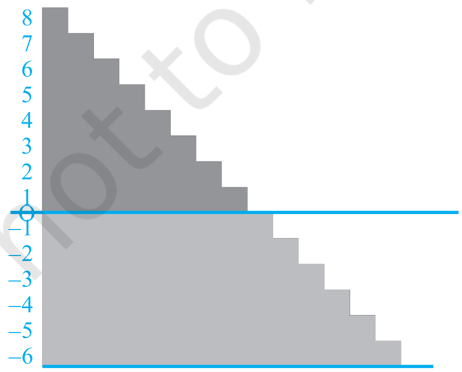
One day Raman and Imran went to the well and counted 8 steps down to water level. They decided to see how much water would come in the well during rains. They marked zero at the existing level of water and marked 1,2,3,4,... above that level for each step. After the rains they noted that the water level rose up to the sixth step. After a few months, they noticed that the water level had fallen three steps below the zero mark. Now, they started thinking about marking the steps to note the fall of water level. Can you help them?

Suddenly, Raman remembered that at one big dam he saw numbers marked even below zero. Imran pointed out that there should be some way to distinguish between numbers which are above zero and below zero. Then Raman recalled that the numbers which were below zero had minus sign in front of them. So they marked one step below zero as – 1 and two steps below zero as – 2 and so on.



So the water level is now at – 3 (3 steps below zero). After that due to further use, the water level went down by 1 step and it was at – 4. You can see that – 4 < – 3.

Keeping in mind the above example, fill in the boxes using > and < signs.



0	☐	– 1	– 100	☐	–101
– 50	☐	– 70	50	☐	–51
– 53	☐	– 5	– 7	☐	1

Let us once again observe the integers which are represented on the number line.

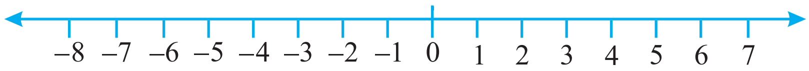


Fig 6.3

We know that 7 > 4 and from the number line shown above, we observe that 7 is to the right of 4 (Fig 6.3).

Similarly, 4 > 0 and 4 is to the right of 0. Now, since 0 is to the right of –3 so, 0 > – 3. Again, – 3 is to the right of – 8 so, – 3 > – 8.

Thus, we see that on a number line the number increases as we move to the right and decreases as we move to the left.

Therefore, – 3 < – 2, – 2 < – 1, – 1 < 0, 0 < 1, 1 < 2, 2 < 3 so on.

Hence, the collection of integers can be written as..., –5, –4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5...

Try These

Compare the following pairs of numbers using > or <.

0 ☐ – 8; – 1 ☐ – 15

5 ☐ – 5; 11 ☐ 15

0 ☐ 6; – 20 ☐ 2

From the above exercise, Rohini arrived at the following conclusions :

(a) Every positive integer is larger than every negative integer.

(b) Zero is less than every positive integer.

(c) Zero is larger than every negative integer.

(d) Zero is neither a negative integer nor a positive integer.

(e) Farther a number from zero on the right, larger is its value.

(f) Farther a number from zero on the left, smaller is its value.

Do you agree with her? Give examples.

Example 1 : By looking at the number line, answer the following questions : Which integers lie between – 8 and – 2? Which is the largest integer and the smallest integer among them?

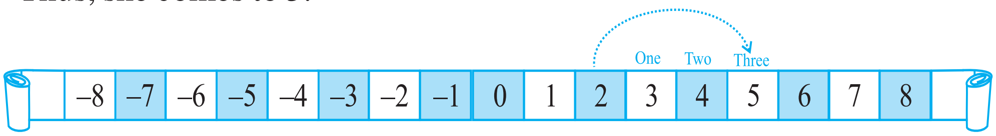
Solution : Integers between – 8 and – 2 are – 7, – 6, – 5, – 4, – 3. The integer – 3 is the largest and – 7 is the smallest.

If, I am not at zero what happens when I move?

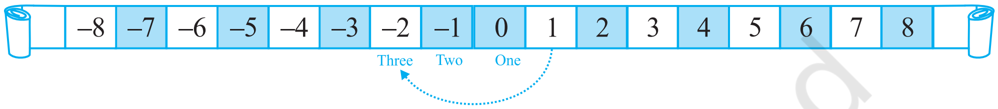
Let us consider the earlier game being played by Salma and Ruchika.

Suppose Ruchika's token is at 2. At the next turn she gets a red die which after throwing gives a number 3. It means she will move 3 places to the right of 2.

Thus, she comes to 5.



If on the other hand, Salma was at 1, and drawn a blue die which gave her number 3, then it means she will move to the left by 3 places and stand at – 2.



By looking at the number line, answer the following question :

Example 2 : (a) One button is kept at – 3. In which direction and how many steps should we move to reach at – 9?

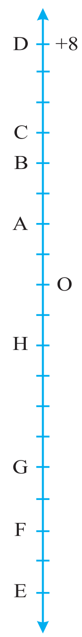
(b) Which number will we reach if we move 4 steps to the right of – 6.

Solution : (a) We have to move six steps to the left of – 3.

(b) We reach – 2 when we move 4 steps to the right of – 6.

EXERCISE 6.1

1. Write opposites of the following :
   (a) Increase in weight (b) 30 km north (c) 80 m east
   (d) Loss of Rs 700 (e) 100 m above sea level
2. Represent the following numbers as integers with appropriate signs.
   (a) An aeroplane is flying at a height two thousand metre above the ground.
   (b) A submarine is moving at a depth, eight hundred metre below the sea level.
   (c) A deposit of rupees two hundred.
   (d) Withdrawal of rupees seven hundred.
3. Represent the following numbers on a number line :
   (a) + 5 (b) – 10 (c) + 8
   (d) – 1 (e) – 6
4. Adjacent figure is a vertical number line, representing integers. Observe it and locate the following points :
   (a) If point D is + 8, then which point is – 8?



(b) Is point G a negative integer or a positive integer?

(c) Write integers for points B and E.

(d) Which point marked on this number line has the least value?

(e) Arrange all the points in decreasing order of value.

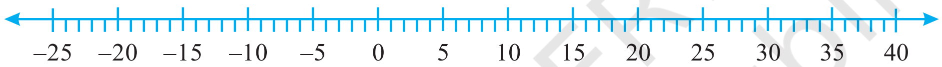
5. Following is the list of temperatures of five places in India on a particular day of the year.

Place	**Temperature**	
Siachin	10°C below 0°C	....
Shimla	2°C below 0°C	....
Ahmedabad	30°C above 0°C	....
Delhi	20°C above 0°C	....
Srinagar	5°C below 0°C	....

(a) Write the temperatures of these places in the form of integers in the blank column.

(b) Following is the number line representing the temperature in degree Celsius.

Plot the name of the city against its temperature.



(c) Which is the coolest place?

(d) Write the names of the places where temperatures are above 10°C.

6. In each of the following pairs, which number is to the right of the other on the number line?

(a) 2, 9 (b) – 3, – 8 (c) 0, – 1

(d) – 11, 10 (e) – 6, 6 (f) 1, – 100

7. Write all the integers between the given pairs (write them in the increasing order.)

(a) 0 and – 7 (b) – 4 and 4

(c) – 8 and – 15 (d) – 30 and – 23

8. (a) Write four negative integers greater than – 20.

(b) Write four integers less than – 10.

9. For the following statements, write True (T) or False (F). If the statement is false, correct the statement.

(a) – 8 is to the right of – 10 on a number line.

(b) – 100 is to the right of – 50 on a number line.

(c) Smallest negative integer is – 1.

(d) – 26 is greater than – 25.

10. Draw a number line and answer the following :
   (a) Which number will we reach if we move 4 numbers to the right of – 2.
   (b) Which number will we reach if we move 5 numbers to the left of 1.
   (c) If we are at – 8 on the number line, in which direction should we move to reach – 13?
   (d) If we are at – 6 on the number line, in which direction should we move to reach – 1?

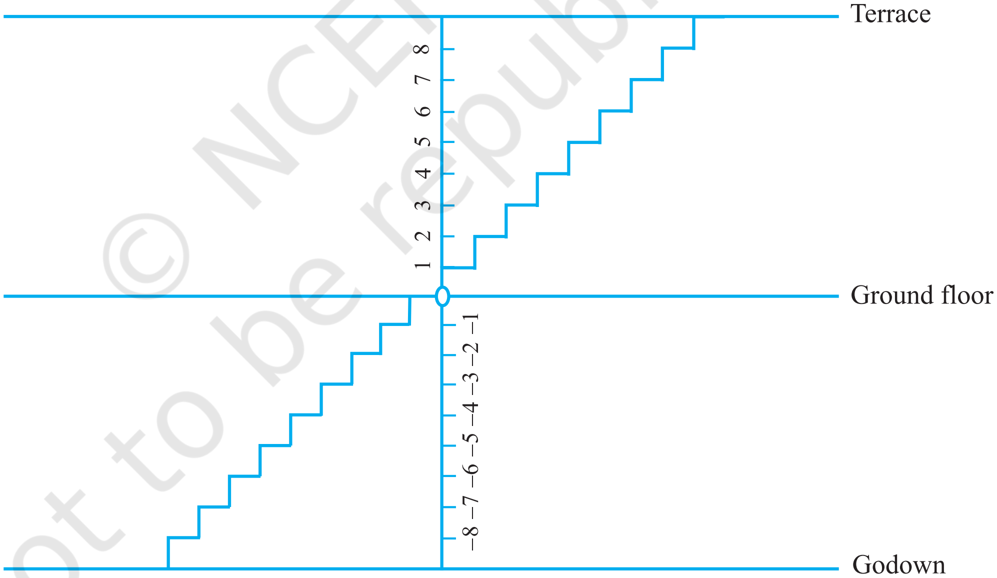
6.3 Additon of Integers

Do This

(Going up and down)

In Mohan's house, there are stairs for going up to the terrace and for going down to the godown.

Let us consider the number of stairs going up to the terrace as positive integer, the number of stairs going down to the godown as negative integer, and the number representing ground level as zero.



Do the following and write down the answer as integer :

(a) Go 6 steps up from the ground floor.
(b) Go 4 steps down from the ground floor.
(c) Go 5 steps up from the ground floor and then go 3 steps up further from there.
(d) Go 6 steps down from the ground floor and then go down further 2 steps from there.

(e) Go down 5 steps from the ground floor and then move up 12 steps from there.

(f) Go 8 steps down from the ground floor and then go up 5 steps from there.

(g) Go 7 steps up from the ground floor and then 10 steps down from there.

Ameena wrote them as follows :

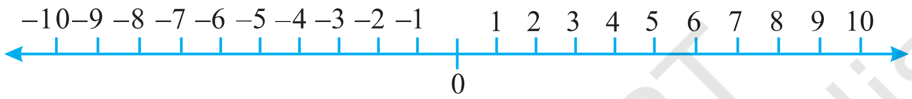
(a) + 6 (b) – 4 (c) (+5) + (+ 3) = + 8 (d) (– 6) + (–2) = – 4

(e) (– 5) + (+12) = + 7 (f) (– 8) + (+5) = – 3 (g) (+7) + (–10) = 17

She has made some mistakes. Can you check her answers and correct those that are wrong?

Try These

Draw a figure on the ground in the form of a horizontal number line as shown below. Frame questions as given in the said example and ask your friends.

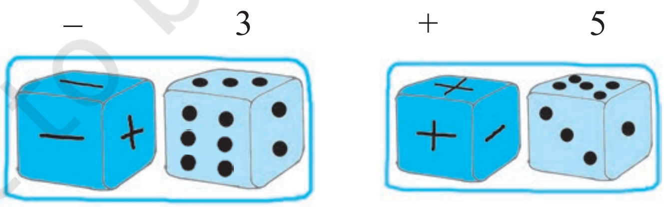


A Game

Take a number strip marked with integers from + 25 to – 25.

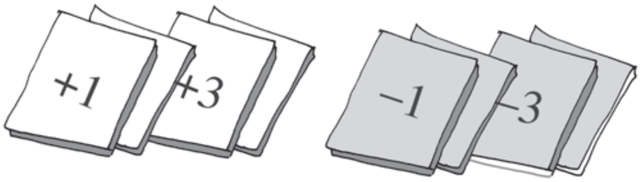
Take two dice, one marked 1 to 6 and the other marked with three '+' signs and three '–' signs.

Players will keep different coloured buttons (or plastic counters) at the zero position on the number strip. In each throw, the player has to see what she has obtained on the two dice. If the first die shows 3 and the second die shows – sign, she has –3. If the first die shows 5 and the second die shows '+' sign, then, she has +5.



Whenever a player gets the + sign, she has to move in the forward direction (towards + 25) and if she gets '–' sign then she has to move in the backward direction (towards – 25).

Each player will throw both dice simultaneously. A player whose counter touches –25 is out of the game and the one whose counter touches + 25 first, wins the game.



You can play the same game with 12 cards marked with + 1, + 2, + 3, + 4, + 5 and + 6 and –1, – 2, ...– 6. Shuffle the cards after every attempt.

Kamla, Reshma and Meenu are playing this game.

Kamla got + 3, + 2, + 6 in three successive attempts. She kept her counter at the mark +11.

Reshma got – 5, + 3, + 1. She kept her counter at – 1. Meenu got + 4, – 3, –2 in three successive attempts; at what position will her counter be? At –1 or at + 1?

Do This

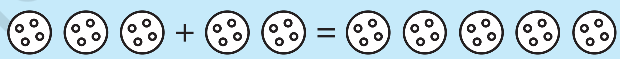
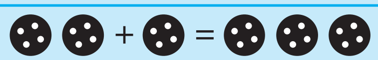
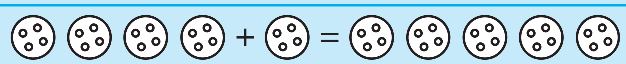
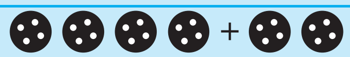
Take two different coloured buttons like white and black. Let us denote one white button by (+ 1) and one black button by (– 1). A pair of one white button (+ 1) and one black button (– 1) will denote zero i.e. [1 + (– 1) = 0]

In the following table, integers are shown with the help of coloured buttons.

Coloured Button	Integers
	5
	– 3
	0

Let us perform additions with the help of the coloured buttons.

Observe the following table and complete it.

	(+ 3) + (+ 2) = +5
	(– 2) + (– 1) = –3
	....
 = ...	....

Try These

Find the answers of the following additions:

(a) (– 11) + (– 12)

(b) (+ 10) + (+ 4)

(c) (– 32) + (– 25)

(d) (+ 23) + (+ 40)

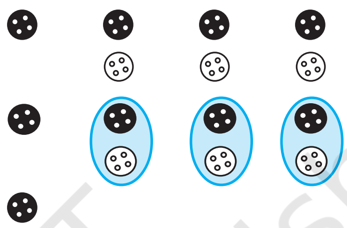
You add when you have two positive integers like (+3) + (+2) = +5 [= 3 + 2]. You also add when you have two negative integers, but the answer will take a minus (–) sign like (–2) + (–1) = – (2+1) = –3.

Now add one positive integer with one negative integer with the help of these buttons. Remove buttons in pairs i.e. a white button with a black button [since (+ 1) + (– 1) = 0]. Check the remaining buttons.

(a) (– 4) + (+ 3)

= (– 1) + (– 3) + (+ 3)

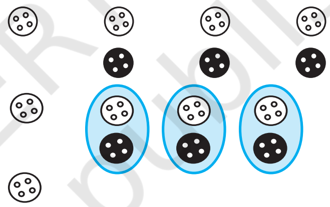
= (– 1) + 0 = – 1



(b) (+ 4) + (– 3)

= (+ 1) + (+ 3) + (– 3)

= (+ 1) + 0 = + 1



You can see that the answer of 4 – 3 is 1 and – 4 + 3 is – 1.

So, when you have one positive and one negative integer, you must subtract, but answer will take the sign of the bigger integer (Ignoring the signs of the numbers decide which is bigger). Some more examples will help :

Try These

Find the solution of the following:

(a) (– 7) + (+ 8)

(b) (– 9) + (+13)

(c) (+ 7) + (– 10)

(d) (+12) + (– 7)

(c) (+ 5) + (– 8) = (+ 5) + (– 5) + (– 3) = 0 + (– 3)
= (– 3)

(d) (+ 6) + (– 4) = (+ 2) + (+ 4) + (– 4) = (+ 2) + 0
= + 2

6.3.1 Addition of integers on a number line

It is not always easy to add integers using coloured buttons. Shall we use number line for additions?

(i) Let us add 3 and 5 on number line.

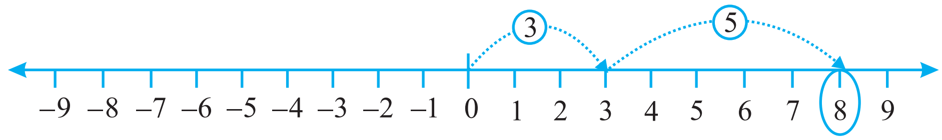


Fig 6.4

On the number line, we first move 3 steps to the right from 0 reaching 3, then we move 5 steps to the right of 3 and reach 8. Thus, we get 3 + 5 = 8 (Fig 6.4)

(ii) Let us add – 3 and – 5 on the number line.

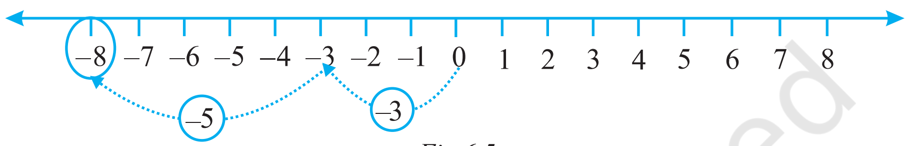


Fig 6.5

On the number line, we first move 3 steps to the left of 0 reaching – 3, then we move 5 steps to the left of – 3 and reach – 8. (Fig 6.5)

Thus, (– 3) + (– 5) = – 8.

We observe that when we add two positive integers, their sum is a positive integer. When we add two negative integers, their sum is a negative integer.

(iii) Suppose we wish to find the sum of (+ 5) and (– 3) on the number line. First we move to the right of 0 by 5 steps reaching 5. Then we move 3 steps to the left of 5 reaching 2. (Fig 6.6)

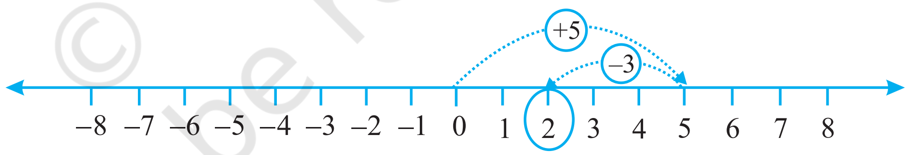


Fig 6.6

Thus, (+ 5) + (– 3) = 2

(iv) Similarly, let us find the sum of (– 5) and (+ 3) on the number line. First we move 5 steps to the left of 0 reaching – 5 and then from this point we move 3 steps to the right. We reach the point – 2.

Thus, (– 5) + (+3) = – 2. (Fig 6.7)

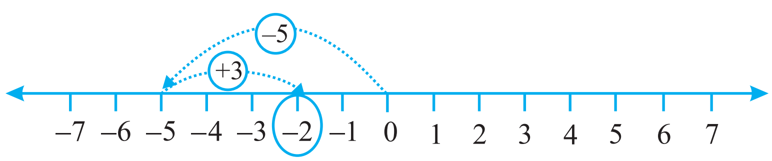


Fig 6.7

Try These

1. Find the solution of the following additions using a number line :

   (a) (– 2) + 6 (b) (– 6) + 2

   Make two such questions and solve them using the number line.

2. Find the solution of the following without using number line :

   (a) (+ 7) + (– 11)

   (b) (– 13) + (+ 10)

   (c) (– 7) + (+ 9)

   (d) (+ 10) + (– 5)

   Make five such questions and solve them.

When a positive integer is added to an integer, the resulting integer becomes greater than the given integer. When a negative integer is added to an integer, the resulting integer becomes less than the given integer.

Let us add 3 and – 3. We first move from 0 to + 3 and then from + 3, we move 3 points to the left. Where do we reach ultimately?

From the Figure 6.8, 3 + (– 3) = 0. Similarly, if we add 2 and – 2, we obtain the sum as zero.

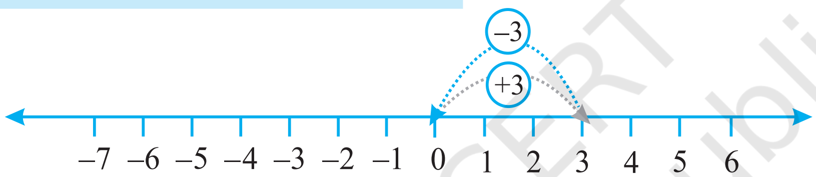


Fig 6.8

Numbers such as 3 and – 3, 2 and – 2, when added to each other give the sum zero. They are called **additive inverse** of each other.

What is the additive inverse of 6? What is the additive inverse of – 7?

Example 3 : Using the number line, write the integer which is

(a) 4 more than –1

(b) 5 less than 3

Solution : (a) We want to know the integer which is 4 more than –1. So, we start from –1 and proceed 4 steps to the right of –1 to reach 3 as shown below:

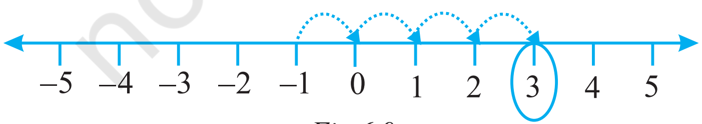


Fig 6.9

Therefore, 4 more than –1 is 3 (Fig 6.9).

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(b) We want to know an integer which is 5 less than 3; so we start from 3 and move to the left by 5 steps and obtain –2 as shown below :

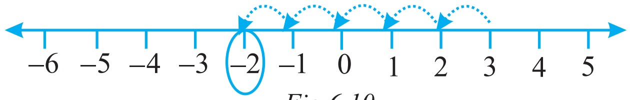


Fig 6.10

Therefore, 5 less than 3 is –2. (Fig 6.10)

Example 4 : Find the sum of (– 9) + (+ 4) + (– 6) + (+ 3)

Solution : We can rearrange the numbers so that the positive integers and the negative integers are grouped together. We have

$$(-9) + (+4) + (-6) + (+3) = (-9) + (-6) + (+4) + (+3) = (-15) + (+7) = -8$$

Example 5 : Find the value of (30) + (– 23) + (– 63) + (+ 55)

Solution : (30) + (+ 55) + (– 23) + (– 63) = 85 + (– 86) = – 1

Example 6 : Find the sum of (– 10), (92), (84) and (– 15)

Solution : (– 10) + (92) + (84) + (– 15) = (– 10) + (– 15) + 92 + 84
= (– 25) + 176 = 151

EXERCISE 6.2

1. Using the number line write the integer which is :
   (a) 3 more than 5
   (b) 5 more than –5
   (c) 6 less than 2
   (d) 3 less than –2

2. Use number line and add the following integers :
   (a) 9 + (– 6)
   (b) 5 + (– 11)
   (c) (– 1) + (– 7)
   (d) (– 5) + 10
   (e) (– 1) + (– 2) + (– 3)
   (f) (– 2) + 8 + (– 4)
3. Add without using number line :
   (a) 11 + (– 7)
   (b) (– 13) + (+ 18)
   (c) (– 10) + (+ 19)
   (d) (– 250) + (+ 150)
   (e) (– 380) + (– 270)
   (f) (– 217) + (– 100)

4. Find the sum of :
   (a) 137 and – 354 (b) – 52 and 52
   (c) – 312, 39 and 192 (d) – 50, – 200 and 300
5. Find the sum :
   (a) (– 7) + (– 9) + 4 + 16
   (b) (37) + (– 2) + (– 65) + (– 8)

6.4 Subtraction of Integers with the help of a Number Line

We have added positive integers on a number line. For example, consider 6+2. We start from 6 and go 2 steps to the right side. We reach at 8. So, 6 + 2 = 8. (Fig 6.11)

Fig 6.11

We also saw that to add 6 and (–2) on a number line we can start from 6 and then move 2 steps to the left of 6. We reach at 4. So, we have, 6 + (–2) = 4. (Fig 6.12)

Fig 6.12

Thus, we find that, to add a positive integer we move towards the right on a number line and for adding a negative integer we move towards left.

We have also seen that while using a number line for whole numbers, for subtracting 2 from 6, we would move towards left. (Fig 6.13)

Fig 6.13

i.e. 6 – 2 = 4

What would we do for 6 – (–2)? Would we move towards the left on the number line or towards the right?

If we move to the left then we reach 4.

Then we have to say 6 – (–2) = 4. This is not true because we know 6 – 2 = 4 and 6 – 2 ≠ 6 – (–2).

So, we have to move towards the right. (Fig 6.14)

Fig 6.14

i.e. 6 – (–2) = 8

This also means that when we subtract a negative integer we get a greater integer. Consider it in another way. We know that additive inverse of (–2) is 2. Thus, it appears that adding the additive inverse of –2 to 6 is the same as subtracting (–2) from 6.

We write 6 – (–2) = 6 + 2.

Let us now find the value of –5 – (–4) using a number line. We can say that this is the same as –5 + (4), as the additive inverse of –4 is 4.

We move 4 steps to the right on the number line starting from –5. (Fig 6.15)

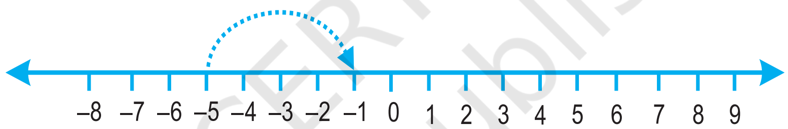


Fig 6.15

We reach at –1.

i.e. –5 + 4 = –1. Thus, –5 – (–4) = –1.

Example 7 : Find the value of – 8 – (–10) using number line

Solution : – 8 – (– 10) is equal to – 8 + 10 as additive inverse of –10 is 10. On the number line, from – 8 we will move 10 steps towards right. (Fig 6.16)

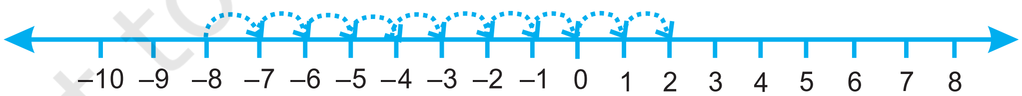


Fig 6.16

We reach at 2. Thus, –8 – (–10) = 2

Hence, to subtract an integer from another integer it is enough to add the additive inverse of the integer that is being subtracted, to the other integer.

Example 8 : Subtract (– 4) from (– 10)

Solution : (– 10) – (– 4) = (– 10) + (additive inverse of – 4)

= –10 + 4 = – 6

Example 9 : Subtract $(+3)$ from (-3)

Solution : $(-3) - (+3) = (-3) + (\text{additive inverse of } +3)$

$= (-3) + (-3) = -6$

EXERCISE 6.3

1. Find

   (a) $35 - (20)$ (b) $72 - (90)$

   (c) $(-15) - (-18)$ (d) $(-20) - (13)$

   (e) $23 - (-12)$ (f) $(-32) - (-40)$

2. Fill in the blanks with >, < or = sign.

   (a) $(-3) + (-6)$ ______ $(-3) - (-6)$

   (b) $(-21) - (-10)$ _____ $(-31) + (-11)$

   (c) $45 - (-11)$ ______ $57 + (-4)$

   (d) $(-25) - (-42)$ _____ $(-42) - (-25)$

3. Fill in the blanks.

   (a) $(-8) +$ _____ $= 0$

   (b) $13 +$ _____ $= 0$

   (c) $12 + (-12) =$ ____

   (d) $(-4) +$ ____ $= -12$

   (e) ____ $-15 = -10$

4. Find

   (a) $(-7) - 8 - (-25)$

   (b) $(-13) + 32 - 8 - 1$

   (c) $(-7) + (-8) + (-90)$

   (d) $50 - (-40) - (-2)$

What have we discussed?

1. We have seen that there are times when we need to use numbers with a negative sign. This is when we want to go below zero on the number line. These are called *negative numbers*. Some examples of their use can be in temperature scale, water level in lake or river, level of oil in tank etc. They are also used to denote debit account or outstanding dues.

2. The collection of numbers..., – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, ... is called *integers*.
   So, – 1, – 2, – 3, – 4, ... called negative numbers are negative integers and 1, 2, 3, 4, ... called positive numbers are the positive integers.
3. We have also seen how one more than given number gives a successor and one less than given number gives predecessor.
4. We observe that
   (a) When we have the same sign, add and put the same sign.
      (i) When two positive integers are added, we get a positive integer [e.g. (+ 3) + (+ 2) = + 5].
      (ii) When two negative integers are added, we get a negative integer [e.g. (–2) + (– 1) = – 3].
   (b) When one positive and one negative integers are added we subtract them as whole numbers by considering the numbers without their sign and then put the sign of the bigger number with the subtraction obtained. The bigger integer is decided by ignoring the signs of the integers [e.g. (+4) + (–3) = + 1 and (–4) + (+ 3) = – 1].
   (c) The subtraction of an integer is the same as the addition of its additive inverse.
5. We have shown how addition and subtraction of integers can also be shown on a number line.

Fractions

Chapter 7

7.1 Introduction

Subhash had learnt about fractions in Classes IV and V, so whenever possible he would try to use fractions. One occasion was when he forgot his lunch at home. His friend Farida invited him to share her lunch. She had five pooris in her lunch box. So, Subhash and Farida took two pooris each. Then Farida made two equal halves of the fifth poori and gave one-half to Subhash and took the other half herself. Thus, both Subhash and Farida had 2 full pooris and one-half poori.

2 pooris + half-poori–Subhash
2 pooris + half-poori–Farida

Where do you come across situations with fractions in your life?

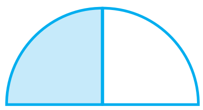


Fig 7.1

Subhash knew that one-half is written as $\frac{1}{2}$. While eating he further divided his half poori into two equal parts and asked Farida what fraction of the whole poori was that piece? (Fig 7.1)

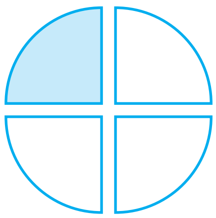


Fig 7.2

Without answering, Farida also divided her portion of the half puri into two equal parts and kept them beside Subhash's shares. She said that these four equal parts together make one whole (Fig 7.2). So, each equal part is one-fourth of one whole poori and 4 parts together will be $\frac{4}{4}$ or 1 whole poori.

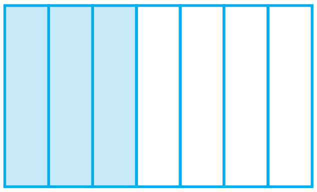


Fig 7.3

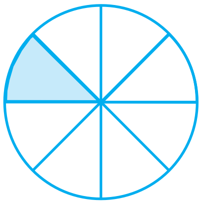


Fig 7.4

When they ate, they discussed what they had learnt earlier. Three parts out of 4 equal parts is $\frac{3}{4}$. Similarly, $\frac{3}{7}$ is obtained when we divide a whole into seven equal parts and take three parts (Fig 7.3). For $\frac{1}{8}$, we divide a whole into eight equal parts and take one part out of it (Fig 7.4).

Farida said that we have learnt that **a fraction is a number representing part of a whole. The whole may be a single object or a group of objects.** Subhash observed that **the parts have to be equal.**

7.2 A Fraction

Let us recapitulate the discussion.

A fraction means a part of a group or of a region.

$\frac{5}{12}$ is a fraction. We read it as "five-twelfths".

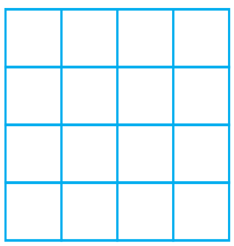
What does "12" stand for? It is the number of equal parts into which the whole has been divided.

What does "5" stand for? It is the number of equal parts which have been taken out.

Here 5 is called the numerator and 12 is called the denominator.

Name the numerator of $\frac{3}{7}$ and the denominator of $\frac{4}{15}$.

Play this Game



You can play this game with your friends.

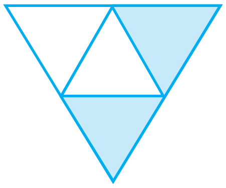
Take many copies of the grid as shown here.

Consider any fraction, say $\frac{1}{2}$.

Each one of you should shade $\frac{1}{2}$ of the grid.

EXERCISE 7.1

1. Write the fraction representing the shaded portion.



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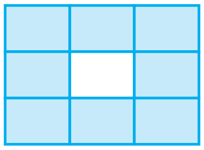

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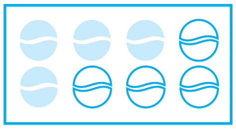

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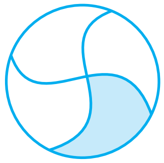

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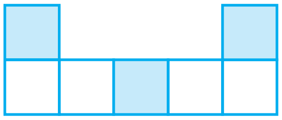

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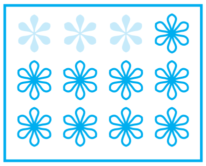

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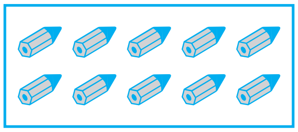

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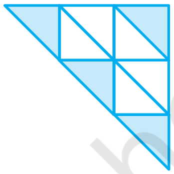

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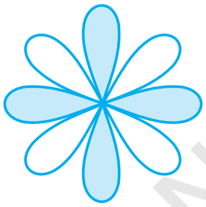

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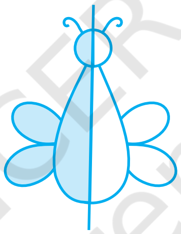
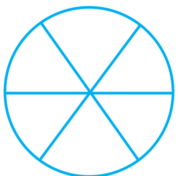
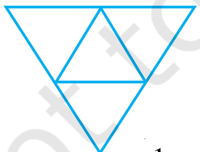


(x)

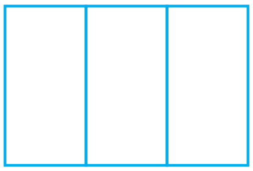
2. Colour the part according to the given fraction.



(i) $\frac{1}{6}$

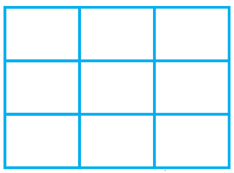


(ii) $\frac{1}{4}$



(iii) $\frac{1}{3}$

(iv) $\frac{3}{4}$

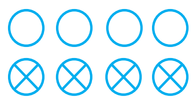


(v) $\frac{4}{9}$

3. Identify the error, if any.

This is $\frac{1}{2}$ This is $\frac{1}{4}$ This is $\frac{3}{4}$

4. What fraction of a day is 8 hours?
5. What fraction of an hour is 40 minutes?
6. Arya, Abhimanyu, and Vivek shared lunch. Arya has brought two sandwiches, one made of vegetable and one of jam. The other two boys forgot to bring their lunch. Arya agreed to share his sandwiches so that each person will have an equal share of each sandwich.
   (a) How can Arya divide his sandwiches so that each person has an equal share?
   (b) What part of a sandwich will each boy receive?
7. Kanchan dyes dresses. She had to dye 30 dresses. She has so far finished 20 dresses. What fraction of dresses has she finished?
8. Write the natural numbers from 2 to 12. What fraction of them are prime numbers?
9. Write the natural numbers from 102 to 113. What fraction of them are prime numbers?
10. What fraction of these circles have X's in them?



11. Kristin received a CD player for her birthday. She bought 3 CDs and received 5 others as gifts. What fraction of her total CDs did she buy and what fraction did she receive as gifts?

7.3 Fraction on the Number Line

You have learnt to show whole numbers like 0,1,2... on a number line.

We can also show fractions on a number line. Let us draw a number line and try to mark $\frac{1}{2}$ on it.

We know that $\frac{1}{2}$ is greater than 0 and less than 1, so it should lie between 0 and 1.

Since we have to show $\frac{1}{2}$, we divide the gap between 0 and 1 into two equal parts and show 1 part as $\frac{1}{2}$ (as shown in the Fig 7.5).

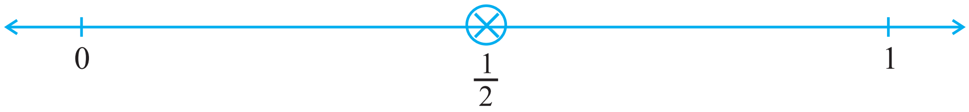


Fig 7.5

Suppose we want to show $\frac{1}{3}$ on a number line. Into how many equal parts should the length between 0 and 1 be divided? We divide the length between 0 and 1 into 3 equal parts and show one part as $\frac{1}{3}$ (as shown in the Fig 7.6)

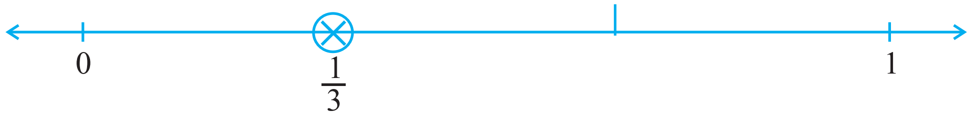


Fig 7.6

Can we show $\frac{2}{3}$ on this number line? $\frac{2}{3}$ means 2 parts out of 3 parts as shown (Fig 7.7).

Fig 7.7

Similarly, how would you show $\frac{0}{3}$ and $\frac{3}{3}$ on this number line?

$\frac{0}{3}$ is the point zero whereas since $\frac{3}{3}$ is 1 whole, it can be shown by the point 1 (as shown in Fig 7.7)

So if we have to show $\frac{3}{7}$ on a number line, then, into how many equal parts should the length between 0 and 1 be divided? If P shows $\frac{3}{7}$ then how many equal divisions lie between 0 and P? Where do $\frac{0}{7}$ and $\frac{7}{7}$ lie?

Try These

1. Show $\frac{3}{5}$ on a number line.
2. Show $\frac{1}{10}, \frac{0}{10}, \frac{5}{10}$ and $\frac{10}{10}$ on a number line.
3. Can you show any other fraction between 0 and 1?
   Write five more fractions that you can show and depict them on the number line.
4. How many fractions lie between 0 and 1? Think, discuss and write your answer?

7.4 Proper Fractions

You have now learnt how to locate fractions on a number line. Locate the fractions $\frac{3}{4}, \frac{1}{2}, \frac{9}{10}, \frac{0}{3}, \frac{5}{8}$ on separate number lines.

Does any one of the fractions lie beyond 1?

All these fractions lie to the left of 1as they are less than 1.

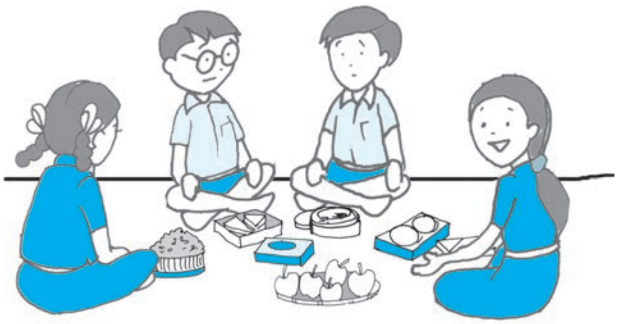
In fact, all the fractions we have learnt so far are less than 1. These are **proper fractions**. A proper fraction as Farida said (Sec. 7.1), is a number representing part of a whole. In a proper fraction the denominator shows the number of parts into which the whole is divided and the numerator shows the number of parts which have been considered. Therefore, in a proper fraction the numerator is always less than the denominator.

Try These

1. Give a proper fraction :
   (a) whose numerator is 5 and denominator is 7.
   (b) whose denominator is 9 and numerator is 5.
   (c) whose numerator and denominator add up to 10. How many fractions of this kind can you make?
   (d) whose denominator is 4 more than the numerator.
   (Give any five. How many more can you make?)
2. A fraction is given.
   How will you decide, by just looking at it, whether, the fraction is
   (a) less than 1?
   (b) equal to 1?
3. Fill up using one of these : '>', '<' or '='
   (a) $\frac{1}{2} \square 1$ (b) $\frac{3}{5} \square 1$ (c) $1 \square \frac{7}{8}$ (d) $\frac{4}{4} \square 1$ (e) $\frac{2005}{2005} \square 1$

7.5 Improper and Mixed Fractions

Anagha, Ravi, Reshma and John shared their tiffin. Along with their food, they had also, brought 5 apples. After eating the other food, the four friends wanted to eat apples.



How can they share five apples among four of them?

Anagha said, 'Let each of us have one full apple and a quarter of the fifth apple.'


Anagha


Ravi


Reshma


John

Reshma said, 'That is fine, but we can also divide each of the five apples into 4 equal parts and take one-quarter from each apple.'


Anagha


Ravi


Reshma


John

Ravi said, 'In both the ways of sharing each of us would get the same share, i.e., 5 quarters. Since 4 quarters make one whole, we can also say that each of us would get 1 whole and one quarter. The value of each share would be five divided by four. Is it written as $5 \div 4$?' John said, 'Yes the same as $\frac{5}{4}$'. Reshma added that in $\frac{5}{4}$, the numerator is bigger than the denominator. The fractions, where the numerator is bigger than the denominator are called **improper fractions.**

Thus, fractions like $\frac{3}{2}, \frac{12}{7}, \frac{18}{5}$ are all improper fractions.

1. Write five improper fractions with denominator 7.
2. Write five improper fractions with numerator 11.

Ravi reminded John, 'What is the other way of writing the share? Does it follow from Anagha's way of dividing 5 apples?'

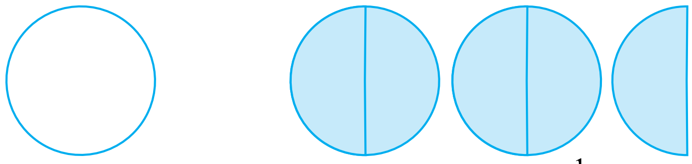
John nodded, 'Yes, It indeed follows from Anagha's way. In her way, each share is one whole and one quarter. It is $1+\frac{1}{4}$ and written in short as $1\frac{1}{4}$. Remember, $1\frac{1}{4}$ is the same as $\frac{5}{4}$'.


This is 1 (one)


Each of these is $\frac{1}{4}$ (one-fourth)

Fig 7.8

Recall the pooris eaten by Farida. She got $2\frac{1}{2}$ poories (Fig 7.9), i.e.



This is 1 This is $2\frac{1}{2}$

Fig 7.9

How many shaded halves are there in $2\frac{1}{2}$? There are 5 shaded halves.

So, the fraction can also be written as $\frac{5}{2}$. $2\frac{1}{2}$ is the same as $\frac{5}{2}$.

Fractions such as $1\frac{1}{4}$ and $2\frac{1}{2}$ are called **Mixed Fractions**. A mixed fraction has a combination of a whole and a part.

> **Do you know?**
> The grip-sizes of tennis racquets are often in mixed numbers. For example one size is '$3\frac{7}{8}$ inches' and '$4\frac{3}{8}$ inches' is another.

Where do you come across mixed fractions? Give some examples.

Example 1 : Express the following as mixed fractions :

(a) $\frac{17}{4}$ (b) $\frac{11}{3}$ (c) $\frac{27}{5}$ (d) $\frac{7}{3}$

Solution : (a) $\frac{17}{4}$

$$4\,\overline{)\,17}\;(4 \qquad 17 - 16 = 1$$

i.e. 4 whole and $\frac{1}{4}$ more, or $4\frac{1}{4}$

(b) $\frac{11}{3}$

$$3\,\overline{)\,11}\;(3 \qquad 11 - 9 = 2$$

i.e. 3 whole and $\frac{2}{3}$ more, or $3\frac{2}{3}$

$$\left[\textit{Alternatively}, \frac{11}{3} = \frac{9+2}{3} = \frac{9}{3} + \frac{2}{3} = 3 + \frac{2}{3} = 3\frac{2}{3}\right]$$

Try (c) and (d) using both the methods for yourself.

Thus, we can express an improper fraction as a mixed fraction by dividing the numerator by denominator to obtain the quotient and the remainder. Then the mixed fraction will be written as $\text{Quotient}\frac{\text{Remainder}}{\text{Divisor}}$.

Example 2 : Express the following mixed fractions as improper fractions:

(a) $2\frac{3}{4}$ (b) $7\frac{1}{9}$ (c) $5\frac{3}{7}$

Solution : (a) $2\frac{3}{4} = 2 + \frac{3}{4} = \frac{2 \times 4}{4} + \frac{3}{4} = \frac{11}{4}$

(b) $7\frac{1}{9} = \frac{(7 \times 9) + 1}{9} = \frac{64}{9}$

(c) $5\frac{3}{7} = \frac{(5 \times 7) + 3}{7} = \frac{38}{7}$

Thus, we can express a mixed fraction as an improper fraction as

$$\frac{(\text{Whole} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}}.$$

EXERCISE 7.2

1. Draw number lines and locate the points on them :

(a) $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{4}{4}$ (b) $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{7}{8}$ (c) $\frac{2}{5}, \frac{3}{5}, \frac{8}{5}, \frac{4}{5}$

2. Express the following as mixed fractions :

(a) $\frac{20}{3}$ (b) $\frac{11}{5}$ (c) $\frac{17}{7}$

(d) $\frac{28}{5}$ (e) $\frac{19}{6}$ (f) $\frac{35}{9}$

3. Express the following as improper fractions :

(a) $7\frac{3}{4}$ (b) $5\frac{6}{7}$ (c) $2\frac{5}{6}$ (d) $10\frac{3}{5}$ (e) $9\frac{3}{7}$ (f) $8\frac{4}{9}$

7.6 Equivalent Fractions

Look at all these representations of fraction (Fig 7.10).

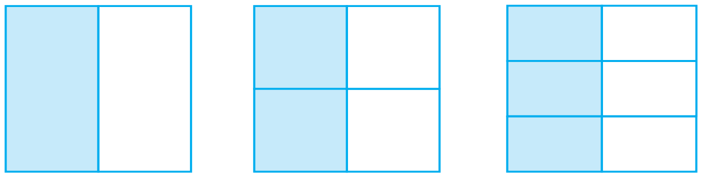
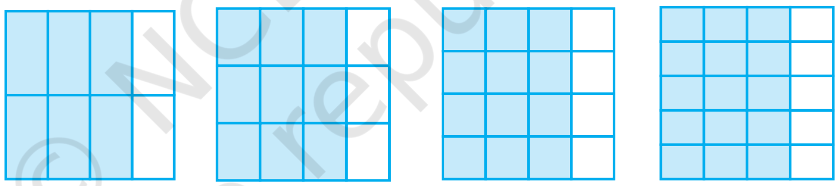


Fig 7.10

These fractions are $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$, representing the parts taken from the total number of parts. If we place the pictorial representation of one over the other they are found to be equal. Do you agree?

Try These

1. Are $\frac{1}{3}$ and $\frac{2}{7}$; $\frac{2}{5}$ and $\frac{2}{7}$; $\frac{2}{9}$ and $\frac{6}{27}$ equivalent? Give reason.
2. Give example of four equivalent fractions.
3. Identify the fractions in each. Are these fractions equivalent?



These fractions are called **equivalent fractions**. Think of three more fractions that are equivalent to the above fractions.

Understanding equivalent fractions

$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, ..., \frac{36}{72}, ...$, are all equivalent fractions. They represent the same part of a whole.

Think, discuss and write

Why do the equivalent fractions represent the same part of a whole? How can we obtain one from the other?

We note $\frac{1}{2} = \frac{2}{4} = \frac{1 \times 2}{2 \times 2}$. Similarly, $\frac{1}{2} = \frac{3}{6} = \frac{1 \times 3}{2 \times 3} = \frac{1}{2}$ and $\frac{1}{2} = \frac{4}{8} = \frac{1 \times 4}{2 \times 4}$

To find an equivalent fraction of a given fraction, you may multiply both the numerator and the denominator of the given fraction by the same number.

Rajni says that equivalent fractions of $\frac{1}{3}$ are :

$\frac{1\times 2}{3\times 2}=\frac{2}{6}$, $\frac{1\times 3}{3\times 3}=\frac{3}{9}$, $\frac{1\times 4}{3\times 4}=\frac{4}{12}$ and many more.

Do you agree with her? Explain.

Try These

1. Find five equivalent fractions of each of the following:

(i) $\frac{2}{3}$ (ii) $\frac{1}{5}$ (iii) $\frac{3}{5}$ (iv) $\frac{5}{9}$

Another way

Is there any other way to obtain equivalent fractions? Look at Fig 7.11.

$\frac{4}{6}$ is shaded here. $\frac{2}{3}$ is shaded here.

Fig 7.11

These include equal number of shaded things i.e. $\frac{4}{6}=\frac{2}{3}=\frac{4\div 2}{6\div 2}$

To find an equivalent fraction, we may divide both the numerator and the denominator by the same number.

One equivalent fraction of $\frac{12}{15}$ is $\frac{12\div 3}{15\div 3}=\frac{4}{5}$

Can you find an equivalent fraction of $\frac{9}{15}$ having denominator 5 ?

Example 3 : Find the equivalent fraction of $\frac{2}{5}$ with numerator 6.

Solution : We know 2 × 3 = 6. This means we need to multiply both the numerator and the denominator by 3 to get the equivalent fraction.

Hence, $\frac{2}{5}=\frac{2\times 3}{5\times 3}=\frac{6}{15}$; $\frac{6}{15}$ is the required equivalent fraction.

Can you show this pictorially?

Example 4 : Find the equivalent fraction of $\frac{15}{35}$ with denominator 7.

Solution : We have $\frac{15}{35}=\frac{\square}{7}$

We observe the denominator and find 35 ÷ 5 = 7. We, therefore, divide both the numerator and the denominator of $\frac{15}{35}$ by 5.

Thus, $\frac{15}{35}=\frac{15\div 5}{35\div 5}=\frac{3}{7}$.

An interesting fact

Let us now note an interesting fact about equivalent fractions. For this, complete the given table. The first two rows have already been completed for you.

Equivalent fractions	Product of the numerator of the 1st and the denominator of the 2nd	Product of the numerator of the 2nd and the denominator of the 1st	Are the products equal?
$\frac{1}{3}=\frac{3}{9}$	$1 \times 9 = 9$	$3 \times 3 = 9$	Yes
$\frac{4}{5}=\frac{28}{35}$	$4 \times 35 = 140$	$5 \times 28 = 140$	Yes
$\frac{1}{4}=\frac{4}{16}$			
$\frac{2}{3}=\frac{10}{15}$			
$\frac{3}{7}=\frac{24}{56}$			

What do we infer? The product of the numerator of the first and the denominator of the second is equal to the product of denominator of the first and the numerator of the second in all these cases. These two products are called cross products. Work out the cross products for other pairs of equivalent fractions. Do you find any pair of fractions for which cross products are not equal? This rule is helpful in finding equivalent fractions.

Example 5 : Find the equivalent fraction of $\frac{2}{9}$ with denominator 63.

Solution : We have $\frac{2}{9} = \frac{\square}{63}$

For this, we should have, $9 \times \square = 2 \times 63$.

But $63 = 7 \times 9$, so $9 \times \square = 2 \times 7 \times 9 = 14 \times 9 = 9 \times 14$

or $9 \times \square = 9 \times 14$

By comparison, $\square = 14$. Therefore, $\frac{2}{9} = \frac{14}{63}$.

7.7 Simplest Form of a Fraction

Given the fraction $\frac{36}{54}$, let us try to get an equivalent fraction in which the numerator and the denominator have no common factor except 1.

How do we do it? We see that both 36 and 54 are divisible by 2.

$$\frac{36}{54} = \frac{36 \div 2}{54 \div 2} = \frac{18}{27}$$

But 18 and 27 also have common factors other than one.
The common factors are 1, 3, 9; the highest is 9.

Therefore, $\frac{18}{27} = \frac{18 \div 9}{27 \div 9} = \frac{2}{3}$

Now 2 and 3 have no common factor except 1; we say that the fraction $\frac{2}{3}$ is in the simplest form.

A fraction is said to be in the simplest (or lowest) form if its numerator and denominator have no common factor except 1.

The shortest way

The shortest way to find the equivalent fraction in the simplest form is to find the HCF of the numerator and denominator, and then divide both of them by the HCF.

A Game

The equivalent fractions given here are quite interesting. Each one of them uses all the digits from 1 to 9 once!

$$\frac{2}{6} = \frac{3}{9} = \frac{58}{174}$$

$$\frac{2}{4} = \frac{3}{6} = \frac{79}{158}$$

Try to find two more such equivalent fractions.

Consider $\frac{36}{24}$.

The HCF of 36 and 24 is 12.

Therefore, $\frac{36}{24} = \frac{36 \div 12}{24 \div 12} = \frac{3}{2}$. The fraction $\frac{3}{2}$ is in the lowest form.

Thus, HCF helps us to reduce a fraction to its lowest form.

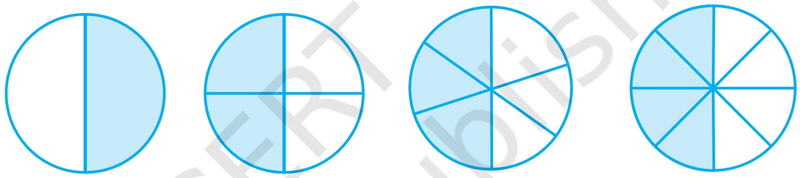
Try These

1. Write the simplest form of :
   (i) $\frac{15}{75}$ (ii) $\frac{16}{72}$
   (iii) $\frac{17}{51}$ (iv) $\frac{42}{28}$ (v) $\frac{80}{24}$
2. Is $\frac{49}{64}$ in its simplest form?

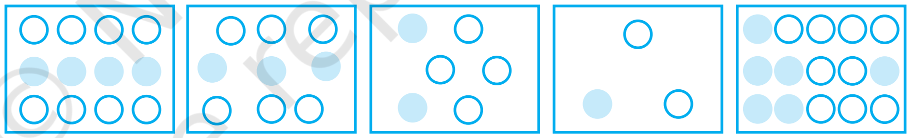
EXERCISE 7.3

1. Write the fractions. Are all these fractions equivalent?

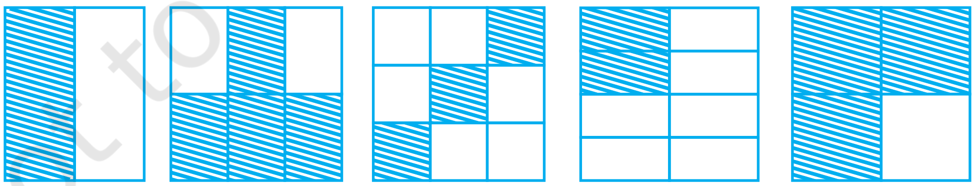
(a)



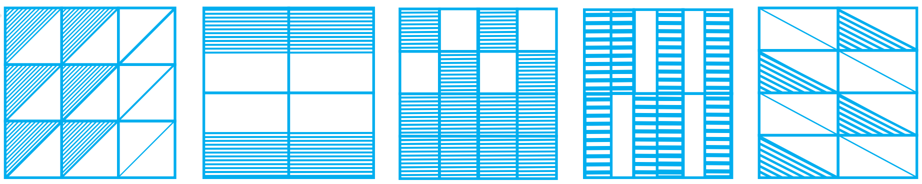
(b)



2. Write the fractions and pair up the equivalent fractions from each row.



(a) (b) (c) (d) (e)



(i) (ii) (iii) (iv) (v)

3. Replace $\square$ in each of the following by the correct number :

(a) $\frac{2}{7} = \frac{8}{\square}$ (b) $\frac{5}{8} = \frac{10}{\square}$ (c) $\frac{3}{5} = \frac{\square}{20}$ (d) $\frac{45}{60} = \frac{15}{\square}$ (e) $\frac{18}{24} = \frac{\square}{4}$

4. Find the equivalent fraction of $\frac{3}{5}$ having

(a) denominator 20 (b) numerator 9

(c) denominator 30 (d) numerator 27

5. Find the equivalent fraction of $\frac{36}{48}$ with

(a) numerator 9 (b) denominator 4

6. Check whether the given fractions are equivalent :

(a) $\frac{5}{9}, \frac{30}{54}$ (b) $\frac{3}{10}, \frac{12}{50}$ (c) $\frac{7}{13}, \frac{5}{11}$

7. Reduce the following fractions to simplest form :

(a) $\frac{48}{60}$ (b) $\frac{150}{60}$ (c) $\frac{84}{98}$ (d) $\frac{12}{52}$ (e) $\frac{7}{28}$

8. Ramesh had 20 pencils, Sheelu had 50 pencils and Jamaal had 80 pencils. After 4 months, Ramesh used up 10 pencils, Sheelu used up 25 pencils and Jamaal used up 40 pencils. What fraction did each use up? Check if each has used up an equal fraction of her/his pencils?

9. Match the equivalent fractions and write two more for each.

(i) $\frac{250}{400}$	(a) $\frac{2}{3}$	(iv) $\frac{180}{360}$	(d) $\frac{5}{8}$
(ii) $\frac{180}{200}$	(b) $\frac{2}{5}$	(v) $\frac{220}{550}$	(e) $\frac{9}{10}$
(iii) $\frac{660}{990}$	(c) $\frac{1}{2}$		

7.8 Like Fractions

Fractions with same denominators are called **like fractions**.

Thus, $\frac{1}{15}, \frac{2}{15}, \frac{3}{15}, \frac{8}{15}$ are all like fractions. Are $\frac{7}{27}$ and $\frac{7}{28}$ like fractions?

Their denominators are different. Therefore, they are not like fractions. They are called **unlike fractions**.

Write five pairs of like fractions and five pairs of unlike fractions.

7.9 Comparing Fractions

Sohni has $3\frac{1}{2}$ rotis in her plate and Rita has $2\frac{3}{4}$ rotis in her plate. Who has more rotis in her plate? Clearly, Sohni has 3 full rotis and more and Rita has less than 3 rotis. So, Sohni has more rotis.

Consider $\frac{1}{2}$ and $\frac{1}{3}$ as shown in Fig. 7.12. The portion of the whole corresponding to $\frac{1}{2}$ is clearly larger than the portion of the same whole corresponding to $\frac{1}{3}$.

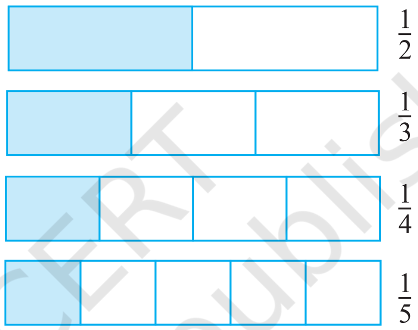


Fig 7.12

So $\frac{1}{2}$ is greater than $\frac{1}{3}$.

But often it is not easy to say which one out of a pair of fractions is larger. For example, which is greater, $\frac{1}{4}$ or $\frac{3}{10}$? For this, we may wish to show the fractions using figures (as in fig. 7.12), but drawing figures may not be easy especially with denominators like 13. We should therefore like to have a systematic procedure to compare fractions. It is particularly easy to compare like fractions. We do this first.

Try These

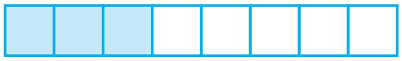
1. You get one-fifth of a bottle of juice and your sister gets one-third of the same size of a bottle of juice. Who gets more?

7.9.1 Comparing like fractions

Like fractions are fractions with the same denominator. Which of these are like fractions?

$\frac{2}{5}, \frac{3}{4}, \frac{1}{5}, \frac{7}{2}, \frac{3}{5}, \frac{4}{5}, \frac{4}{7}$

Let us compare two like fractions: $\frac{3}{8}$ and $\frac{5}{8}$.


$\frac{3}{8}$


$\frac{5}{8}$

In both the fractions the whole is divided into 8 equal parts. For $\frac{3}{8}$ and $\frac{5}{8}$, we take 3 and 5 parts respectively out of the 8 equal parts. Clearly, out of 8 equal parts, the portion corresponding to 5 parts is larger than the portion corresponding to 3 parts. Hence, $\frac{5}{8} > \frac{3}{8}$. Note the number of the parts taken is given by the numerator. It is, therefore, clear that for two fractions with the same denominator, the fraction with the greater numerator is greater. Between $\frac{4}{5}$ and $\frac{3}{5}$, $\frac{4}{5}$ is greater. Between $\frac{11}{20}$ and $\frac{13}{20}$, $\frac{13}{20}$ is greater and so on.

Try These

1. Which is the larger fraction?

   (i) $\frac{7}{10}$ or $\frac{8}{10}$ (ii) $\frac{11}{24}$ or $\frac{13}{24}$ (iii) $\frac{17}{102}$ or $\frac{12}{102}$

   Why are these comparisons easy to make?

2. Write these in ascending and also in descending order.

   (a) $\frac{1}{8}, \frac{5}{8}, \frac{3}{8}$ (b) $\frac{1}{5}, \frac{11}{5}, \frac{4}{5}, \frac{3}{5}, \frac{7}{5}$ (c) $\frac{1}{7}, \frac{3}{7}, \frac{13}{7}, \frac{11}{7}, \frac{7}{7}$

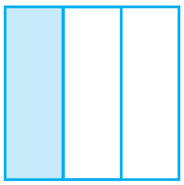
7.9.2 Comparing unlike fractions

Two fractions are unlike if they have different denominators. For example, $\frac{1}{3}$ and $\frac{1}{5}$ are unlike fractions. So are $\frac{2}{3}$ and $\frac{3}{5}$.

Unlike fractions with the same numerator :

Consider a pair of unlike fractions $\frac{1}{3}$ and $\frac{1}{5}$, in which the numerator is the same.

Which is greater $\frac{1}{3}$ or $\frac{1}{5}$?

$\frac{1}{3}$


$\frac{1}{5}$


In $\frac{1}{3}$, we divide the whole into 3 equal parts and take one. In $\frac{1}{5}$, we divide the whole into 5 equal parts and take one. Note that in $\frac{1}{3}$, the whole is divided into a smaller number of parts than in $\frac{1}{5}$. The equal part that we get in $\frac{1}{3}$ is, therefore, larger than the equal part we get in $\frac{1}{5}$. Since in both cases we take the same number of parts (i.e. one), the portion of the whole showing $\frac{1}{3}$ is larger than the portion showing $\frac{1}{5}$, and therfore $\frac{1}{3} > \frac{1}{5}$.

In the same way we can say $\frac{2}{3} > \frac{2}{5}$. In this case, the situation is the same as in the case above, except that the common numerator is 2, not 1. The whole is divided into a large number of equal parts for $\frac{2}{5}$ than for $\frac{2}{3}$. Therefore, each equal part of the whole in case of $\frac{2}{3}$ is larger than that in case of $\frac{2}{5}$. Therefore, the portion of the whole showing $\frac{2}{3}$ is larger than the portion showing $\frac{2}{5}$ and hence, $\frac{2}{3} > \frac{2}{5}$.

We can see from the above example that **if the numerator is the same in two fractions, the fraction with the smaller denominator is greater of the two**.

Thus, $\frac{1}{8} > \frac{1}{10}$, $\frac{3}{5} > \frac{3}{7}$, $\frac{4}{9} > \frac{4}{11}$ and so on.

Let us arrange $\frac{2}{1}, \frac{2}{13}, \frac{2}{9}, \frac{2}{5}, \frac{2}{7}$ in increasing order. All these fractions are unlike, but their numerator is the same. Hence, in such case, the larger the denominator, the smaller is the fraction. The smallest is $\frac{2}{13}$, as it has the largest denominator. The next three fractions in order are $\frac{2}{9}, \frac{2}{7}, \frac{2}{5}$. The greatest fraction is $\frac{2}{1}$ (It is with the smallest denominator). The arrangement in increasing order, therefore, is $\frac{2}{13}, \frac{2}{9}, \frac{2}{7}, \frac{2}{5}, \frac{2}{1}$.

Try These

1. Arrange the following in ascending and descending order :

   (a) $\frac{1}{12}, \frac{1}{23}, \frac{1}{5}, \frac{1}{7}, \frac{1}{50}, \frac{1}{9}, \frac{1}{17}$

   (b) $\frac{3}{7}, \frac{3}{11}, \frac{3}{5}, \frac{3}{2}, \frac{3}{13}, \frac{3}{4}, \frac{3}{17}$

   (c) Write 3 more similar examples and arrange them in ascending and descending order.

Suppose we want to compare $\frac{2}{3}$ and $\frac{3}{4}$. Their numerators are different and so are their denominators. We know how to compare like fractions, i.e. fractions with the same denominator. We should, therefore, try to change the denominators of the given fractions, so that they become equal. For this purpose, we can use the method of equivalent fractions which we already know. Using this method we can change the denominator of a fraction without changing its value.

Let us find equivalent fractions of both $\frac{2}{3}$ and $\frac{3}{4}$.

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = ...$ Similarly, $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = ...$

The equivalent fractions of $\frac{2}{3}$ and $\frac{3}{4}$ with the same denominator 12 are $\frac{8}{12}$ and $\frac{9}{12}$ repectively.

i.e. $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$. Since, $\frac{9}{12} > \frac{8}{12}$ we have, $\frac{3}{4} > \frac{2}{3}$.

Example 6 : Compare $\frac{4}{5}$ and $\frac{5}{6}$.

Solution : The fractions are unlike fractions. Their numerators are different too. Let us write their equivalent fractions.

$$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} = \frac{28}{35} = ...$$

and $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36} = ...$

The equivalent fractions with the same denominator are :

$$\frac{4}{5}=\frac{24}{30} \text{ and } \frac{5}{6}=\frac{25}{30}$$

Since, $\frac{25}{30}>\frac{24}{30}$ so, $\frac{5}{6}>\frac{4}{5}$

Note that the common denominator of the equivalent fractions is 30 which is 5×6. It is a common multiple of both 5 and 6.

So, when we compare two unlike fractions, we first get their equivalent fractions with a denominator which is a common multiple of the denominators of both the fractions.

Example 7 : Compare $\frac{5}{6}$ and $\frac{13}{15}$.

Solution : The fractions are unlike. We should first get their equivalent fractions with a denominator which is a common multiple of 6 and 15.

$$\text{Now, } \frac{5\times5}{6\times5}=\frac{25}{30}, \quad \frac{13\times2}{15\times2}=\frac{26}{30}$$

$$\text{Since } \frac{26}{30}>\frac{25}{30} \text{ we have } \frac{13}{15}>\frac{5}{6}.$$

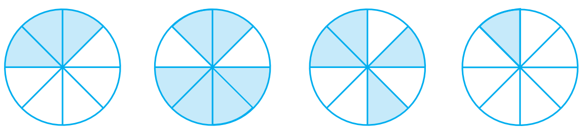
Why LCM?

The product of 6 and 15 is 90; obviously 90 is also a common multiple of 6 and 15. We may use 90 instead of 30; it will not be wrong. But we know that it is easier and more convenient to work with smaller numbers. So the common multiple that we take is as small as possible. This is why the LCM of the denominators of the fractions is preferred as the common denominator.

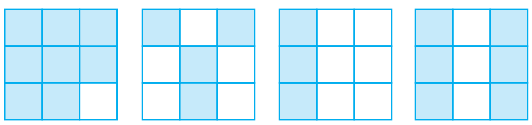
EXERCISE 7.4

1. Write shaded portion as fraction. Arrange them in ascending and descending order using correct sign '<', '=', '>' between the fractions:

(a)



(b)



(c) Show $\frac{2}{6}$, $\frac{4}{6}$, $\frac{8}{6}$ and $\frac{6}{6}$ on the number line. Put appropriate signs between the fractions given.

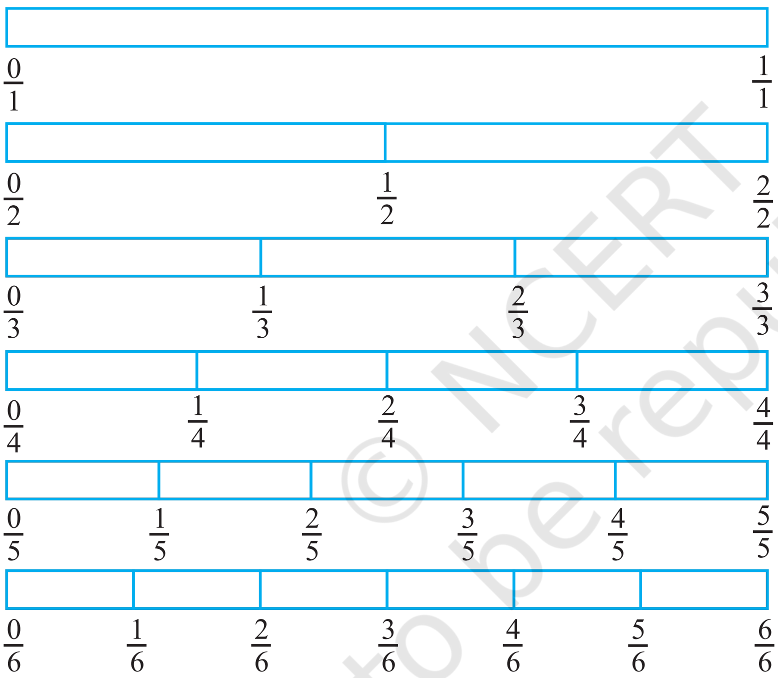
$\frac{5}{6} \square \frac{2}{6}$, $\quad \frac{3}{6} \square 0$, $\quad \frac{1}{6} \square \frac{6}{6}$, $\quad \frac{8}{6} \square \frac{5}{6}$

2. Compare the fractions and put an appropriate sign.

(a) $\frac{3}{6} \square \frac{5}{6}$ (b) $\frac{1}{7} \square \frac{1}{4}$ (c) $\frac{4}{5} \square \frac{5}{5}$ (d) $\frac{3}{5} \square \frac{3}{7}$

3. Make five more such pairs and put appropriate signs.

4. Look at the figures and write '<' or '>', '=' between the given pairs of fractions.



(a) $\frac{1}{6} \square \frac{1}{3}$ (b) $\frac{3}{4} \square \frac{2}{6}$ (c) $\frac{2}{3} \square \frac{2}{4}$ (d) $\frac{6}{6} \square \frac{3}{3}$ (e) $\frac{5}{6} \square \frac{5}{5}$

Make five more such problems and solve them with your friends.

5. How quickly can you do this? Fill appropriate sign. ('<', '=', '>')

(a) $\frac{1}{2} \square \frac{1}{5}$ (b) $\frac{2}{4} \square \frac{3}{6}$ (c) $\frac{3}{5} \square \frac{2}{3}$

(d) $\frac{3}{4} \square \frac{2}{8}$ (e) $\frac{3}{5} \square \frac{6}{5}$ (f) $\frac{7}{9} \square \frac{3}{9}$

(g) $\frac{1}{4} \square \frac{2}{8}$ (h) $\frac{6}{10} \square \frac{4}{5}$ (i) $\frac{3}{4} \square \frac{7}{8}$

(j) $\frac{6}{10} \square \frac{3}{5}$ (k) $\frac{5}{7} \square \frac{15}{21}$

6. The following fractions represent just three different numbers. Separate them into three groups of equivalent fractions, by changing each one to its simplest form.

(a) $\frac{2}{12}$ (b) $\frac{3}{15}$ (c) $\frac{8}{50}$ (d) $\frac{16}{100}$ (e) $\frac{10}{60}$ (f) $\frac{15}{75}$

(g) $\frac{12}{60}$ (h) $\frac{16}{96}$ (i) $\frac{12}{75}$ (j) $\frac{12}{72}$ (k) $\frac{3}{18}$ (l) $\frac{4}{25}$

7. Find answers to the following. Write and indicate how you solved them.

(a) Is $\frac{5}{9}$ equal to $\frac{4}{5}$? (b) Is $\frac{9}{16}$ equal to $\frac{5}{9}$?

(c) Is $\frac{4}{5}$ equal to $\frac{16}{20}$? (d) Is $\frac{1}{15}$ equal to $\frac{4}{30}$?

8. Ila read 25 pages of a book containing 100 pages. Lalita read $\frac{2}{5}$ of the same book. Who read less?

9. Rafiq exercised for $\frac{3}{6}$ of an hour, while Rohit exercised for $\frac{3}{4}$ of an hour. Who exercised for a longer time?

10. In a class A of 25 students, 20 passed with 60% or more marks; in another class B of 30 students, 24 passed with 60% or more marks. In which class was a greater fraction of students getting with 60% or more marks?

7.10 Addition and Subtraction of Fractions

So far in our study we have learnt about natural numbers, whole numbers and then integers. In the present chapter, we are learning about fractions, a different type of numbers.

Whenever we come across new type of numbers, we want to know how to operate with them. Can we combine and add them? If so, how? Can we take away some number from another? i.e., can we subtract one from the other? and so on. Which of the properties learnt earlier about the numbers hold now? Which are the new properties? We also see how these help us deal with our daily life situations.

Try These

1. My mother divided an apple into 4 equal parts. She gave me two parts and my brother one part. How much apple did she give to both of us together?
2. Mother asked Neelu and her brother to pick stones from the wheat. Neelu picked one fourth of the total stones in it and her brother also picked up one fourth of the stones. What fraction of the stones did both pick up together?
3. Sohan was putting covers on his note books. He put one fourth of the covers on Monday. He put another one fourth on Tuesday and the remaining on Wednesday. What fraction of the covers did he put on Wednesday?

Look at the following examples: A tea stall owner consumes in her shop $2\frac{1}{2}$ litres of milk in the morning and $1\frac{1}{2}$ litres of milk in the evening in preparing tea. What is the total amount of milk she uses in the stall?

Or Shekhar ate 2 chapatis for lunch and $1\frac{1}{2}$ chapatis for dinner. What is the total number of chapatis he ate?

Clearly, both the situations require the fractions to be added. Some of these additions can be done orally and the sum can be found quite easily.

Do This

Make five such problems with your friends and solve them.

7.10.1 Adding or subtracting like fractions

All fractions cannot be added orally. We need to know how they can be added in different situations and learn the procedure for it. We begin by looking at addition of like fractions.

Take a 7 × 4 grid sheet (Fig 7.13). The sheet has seven boxes in each row and four boxes in each column.

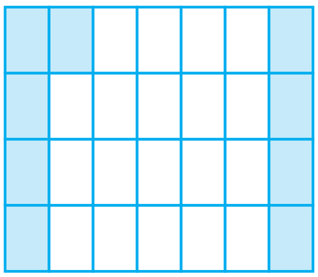


Fig 7.13

How many boxes are there in total?

Colour five of its boxes in green.

What fraction of the whole is the green region?

Now colour another four of its boxes in yellow.

What fraction of the whole is this yellow region?

What fraction of the whole is coloured altogether?

Does this explain that $\frac{5}{28} + \frac{4}{28} = \frac{9}{28}$?

Look at more examples

In Fig 7.14 (i) we have 2 quarter parts of the figure shaded. This means we have 2 parts out of 4 shaded or $\frac{1}{2}$ of the figure shaded.

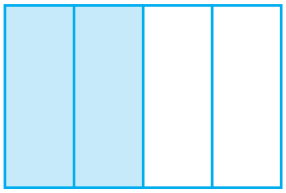


Fig. 7.14 (i)

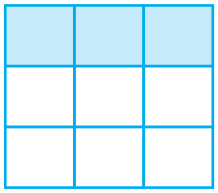


Fig. 7.14 (ii)

That is, $\frac{1}{4}+\frac{1}{4}=\frac{1+1}{4}=\frac{2}{4}=\frac{1}{2}$.

Look at Fig 7.14 (ii)

Fig 7.14 (ii) demonstrates $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}=\frac{1+1+1}{9}=\frac{3}{9}=\frac{1}{3}$.

What do we learn from the above examples? The sum of two or more like fractions can be obtained as follows :

Step 1 Add the numerators.

Step 2 Retain the (common) denominator.

Step 3 Write the fraction as : $\frac{\text{Result of Step 1}}{\text{Result of Step 2}}$

Try These

1. Add with the help of a diagram.
   (i) $\frac{1}{8}+\frac{1}{8}$ (ii) $\frac{2}{5}+\frac{3}{5}$ (iii) $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$
2. Add $\frac{1}{12}+\frac{1}{12}$. How will we show this pictorially? Using paper folding?
3. Make 5 more examples of problems given in 1 and 2 above.
   Solve them with your friends.

Let us, thus, add $\frac{3}{5}$ and $\frac{1}{5}$.

We have $\frac{3}{5}+\frac{1}{5}=\frac{3+1}{5}=\frac{4}{5}$

So, what will be the sum of $\frac{7}{12}$ and $\frac{3}{12}$?

Finding the balance

Sharmila had $\frac{5}{6}$ of a cake. She gave $\frac{2}{6}$ out of that to her younger brother. How much cake is left with her?

A diagram can explain the situation (Fig 7.15). (Note that, here the given fractions are like fractions).

We find that $\frac{5}{6}-\frac{2}{6}=\frac{5-2}{6}=\frac{3}{6}$ or $\frac{1}{2}$

(Is this not similar to the method of adding like fractions?)

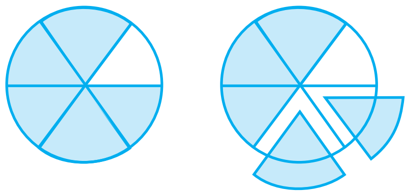


Fig 7.15

Thus, we can say that the difference of two like fractions can be obtained as follows:

Step 1 Subtract the smaller numerator from the bigger numerator.

Step 2 Retain the (common) denominator.

Step 3 Write the fraction as : $\dfrac{\text{Result of Step 1}}{\text{Result of Step 2}}$

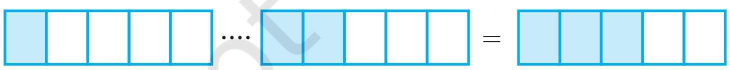
Can we now subtract $\frac{3}{10}$ from $\frac{8}{10}$?

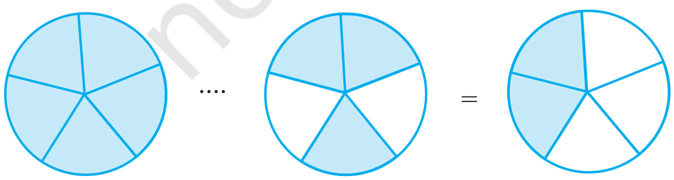
Try These

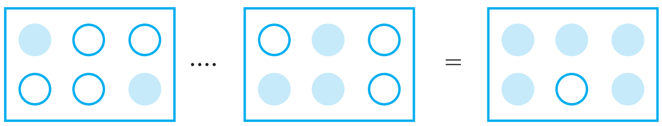
1. Find the difference between $\frac{7}{8}$ and $\frac{3}{8}$.
2. Mother made a gud patti in a round shape. She divided it into 5 parts. Seema ate one piece from it. If I eat another piece then how much would be left?
3. My elder sister divided the watermelon into 16 parts. I ate 7 out them. My friend ate 4. How much did we eat between us? How much more of the watermelon did I eat than my friend? What portion of the watermelon remained?
4. Make five problems of this type and solve them with your friends.

EXERCISE 7.5

1. Write these fractions appropriately as additions or subtractions :

(a) 

(b) 

(c) 

2. Solve :

(a) $\frac{1}{18}+\frac{1}{18}$ (b) $\frac{8}{15}+\frac{3}{15}$ (c) $\frac{7}{7}-\frac{5}{7}$ (d) $\frac{1}{22}+\frac{21}{22}$ (e) $\frac{12}{15}-\frac{7}{15}$

(f) $\frac{5}{8}+\frac{3}{8}$ (g) $1-\frac{2}{3}\left(1=\frac{3}{3}\right)$ (h) $\frac{1}{4}+\frac{0}{4}$ (i) $3-\frac{12}{5}$

3. Shubham painted $\frac{2}{3}$ of the wall space in his room. His sister Madhavi helped and painted $\frac{1}{3}$ of the wall space. How much did they paint together?

4. Fill in the missing fractions.

(a) $\frac{7}{10}-\square=\frac{3}{10}$ (b) $\square-\frac{3}{21}=\frac{5}{21}$ (c) $\square-\frac{3}{6}=\frac{3}{6}$ (d) $\square+\frac{5}{27}=\frac{12}{27}$

5. Javed was given $\frac{5}{7}$ of a basket of oranges. What fraction of oranges was left in the basket?

7.10.2 Adding and subtracting fractions

We have learnt to add and subtract like fractions. It is also not very difficult to add fractions that do not have the same denominator. When we have to add or subtract fractions we first find equivalent fractions with the same denominator and then proceed.

What added to $\frac{1}{5}$ gives $\frac{1}{2}$? This means subtract $\frac{1}{5}$ from $\frac{1}{2}$ to get the required number.

Since $\frac{1}{5}$ and $\frac{1}{2}$ are unlike fractions, in order to subtract them, we first find their equivalent fractions with the same denominator. These are $\frac{2}{10}$ and $\frac{5}{10}$ respectively.

This is because $\frac{1}{2}=\frac{1\times5}{2\times5}=\frac{5}{10}$ and $\frac{1}{5}=\frac{1\times2}{5\times2}=\frac{2}{10}$

Therefore, $\frac{1}{2}-\frac{1}{5}=\frac{5}{10}-\frac{2}{10}=\frac{5-2}{10}=\frac{3}{10}$

Note that 10 is the least common multiple (LCM) of 2 and 5.

Example 8 : Subtract $\frac{3}{4}$ from $\frac{5}{6}$.

Solution : We need to find equivalent fractions of $\frac{3}{4}$ and $\frac{5}{6}$, which have the same denominator. This denominator is given by the LCM of 4 and 6. The required LCM is 12.

Therefore, $\frac{5}{6}-\frac{3}{4}=\frac{5\times2}{6\times2}-\frac{3\times3}{4\times3}=\frac{10}{12}-\frac{9}{12}=\frac{1}{12}$

Example 9 : Add $\frac{2}{5}$ to $\frac{1}{3}$.

Solution : The LCM of 5 and 3 is 15.

Therefore, $\frac{2}{5}+\frac{1}{3}=\frac{2\times3}{5\times3}+\frac{1\times5}{3\times5}=\frac{6}{15}+\frac{5}{15}=\frac{11}{15}$

Example 10 : Simplify $\frac{3}{5}-\frac{7}{20}$

Solution : The LCM of 5 and 20 is 20.

Therefore, $\frac{3}{5}-\frac{7}{20}=\frac{3\times4}{5\times4}-\frac{7}{20}=\frac{12}{20}-\frac{7}{20}$

$=\frac{12-7}{20}=\frac{5}{20}=\frac{1}{4}$

Try These

1. Add $\frac{2}{5}$ and $\frac{3}{7}$.
2. Subtract $\frac{2}{5}$ from $\frac{5}{7}$.

How do we add or subtract mixed fractions?

Mixed fractions can be written either as a whole part plus a proper fraction or entirely as an improper fraction. One way to add (or subtract) mixed fractions is to do the operation seperately for the whole parts and the other way is to write the mixed fractions as improper fractions and then directly add (or subtract) them.

Example 11 : Add $2\frac{4}{5}$ and $3\frac{5}{6}$

Solution : $2\frac{4}{5}+3\frac{5}{6}=2+\frac{4}{5}+3+\frac{5}{6}=5+\frac{4}{5}+\frac{5}{6}$

Now $\frac{4}{5}+\frac{5}{6}=\frac{4\times6}{5\times6}+\frac{5\times5}{6\times5}$ (Since LCM of 5 and 6 = 30)

$=\frac{24}{30}+\frac{25}{30}=\frac{49}{30}=\frac{30+19}{30}=1+\frac{19}{30}$

Thus, $5+\frac{4}{5}+\frac{5}{6}=5+1+\frac{19}{30}=6+\frac{19}{30}=6\frac{19}{30}$

And, therefore, $2\frac{4}{5}+3\frac{5}{6}=6\frac{19}{30}$

Think, discuss and write

Can you find the other way of doing this sum?

Example 12 : Find $4\frac{2}{5} - 2\frac{1}{5}$

Solution : The whole numbers 4 and 2 and the fractional numbers $\frac{2}{5}$ and $\frac{1}{5}$ can be subtracted separately. (Note that 4 > 2 and $\frac{2}{5} > \frac{1}{5}$)

$$\text{So, } 4\frac{2}{5} - 2\frac{1}{5} = (4-2) + \left(\frac{2}{5} - \frac{1}{5}\right) = 2 + \frac{1}{5} = 2\frac{1}{5}$$

Example 13 : Simplify: $8\frac{1}{4} - 2\frac{5}{6}$

Solution : Here 8 > 2 but $\frac{1}{4} < \frac{5}{6}$. We proceed as follows:

$$8\frac{1}{4} = \frac{(8\times4)+1}{4} = \frac{33}{4} \text{ and } 2\frac{5}{6} = \frac{2\times6+5}{6} = \frac{17}{6}$$

$$\text{Now, } \frac{33}{4} - \frac{17}{6} = \frac{33\times3}{12} - \frac{17\times2}{12} \quad \text{(Since LCM of 4 and 6 = 12)}$$

$$= \frac{99-34}{12} = \frac{65}{12} = 5\frac{5}{12}$$

EXERCISE 7.6

1. Solve

(a) $\frac{2}{3} + \frac{1}{7}$ (b) $\frac{3}{10} + \frac{7}{15}$ (c) $\frac{4}{9} + \frac{2}{7}$ (d) $\frac{5}{7} + \frac{1}{3}$ (e) $\frac{2}{5} + \frac{1}{6}$

(f) $\frac{4}{5} + \frac{2}{3}$ (g) $\frac{3}{4} - \frac{1}{3}$ (h) $\frac{5}{6} - \frac{1}{3}$ (i) $\frac{2}{3} + \frac{3}{4} + \frac{1}{2}$ (j) $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

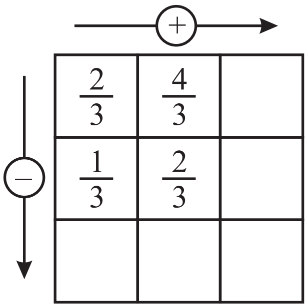
(k) $1\frac{1}{3} + 3\frac{2}{3}$ (l) $4\frac{2}{3} + 3\frac{1}{4}$ (m) $\frac{16}{5} - \frac{7}{5}$ (n) $\frac{4}{3} - \frac{1}{2}$

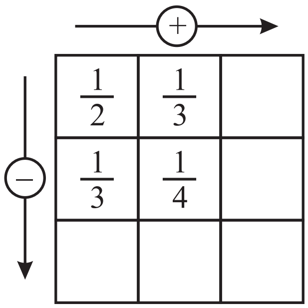
2. Sarita bought $\frac{2}{5}$ metre of ribbon and Lalita $\frac{3}{4}$ metre of ribbon. What is the total length of the ribbon they bought?

3. Naina was given $1\frac{1}{2}$ piece of cake and Najma was given $1\frac{1}{3}$ piece of cake. Find the total amount of cake was given to both of them.

4. Fill in the boxes : (a) $\square - \frac{5}{8} = \frac{1}{4}$ (b) $\square - \frac{1}{5} = \frac{1}{2}$ (c) $\frac{1}{2} - \square = \frac{1}{6}$

5. Complete the addition-subtraction box.

(a) 

(b) 

6. A piece of wire $\frac{7}{8}$ metre long broke into two pieces. One piece was $\frac{1}{4}$ metre long. How long is the other piece?

7. Nandini's house is $\frac{9}{10}$ km from her school. She walked some distance and then took a bus for $\frac{1}{2}$ km to reach the school. How far did she walk?

8. Asha and Samuel have bookshelves of the same size partly filled with books. Asha's shelf is $\frac{5}{6}$th full and Samuel's shelf is $\frac{2}{5}$th full. Whose bookshelf is more full? By what fraction?

9. Jaidev takes $2\frac{1}{5}$ minutes to walk across the school ground. Rahul takes $\frac{7}{4}$ minutes to do the same. Who takes less time and by what fraction?

What have we discussed?

1. (a) A fraction is a number representing a part of a whole. The whole may be a single object or a group of objects.
   (b) When expressing a situation of counting parts to write a fraction, it must be ensured that all parts are equal.
2. In $\frac{5}{7}$, 5 is called the numerator and 7 is called the denominator.
3. Fractions can be shown on a number line. Every fraction has a point associated with it on the number line.
4. In a proper fraction, the numerator is less than the denominator. The fractions, where the numerator is greater than the denominator are called improper fractions. An improper fraction can be written as a combination of a whole and a part, and such fraction then called mixed fractions.
5. Each proper or improper fraction has many equivalent fractions. To find an equivalent fraction of a given fraction, we may multiply or divide both the numerator and the denominator of the given fraction by the same number.
6. A fraction is said to be in the simplest (or lowest) form if its numerator and the denominator have no common factor except 1.

Decimals

Chapter 8

8.1 Introduction

Savita and Shama were going to market to buy some stationary items. Savita said, “I have 5 rupees and 75 paise”. Shama said, “I have 7 rupees and 50 paise”.

They knew how to write rupees and paise using decimals.

So Savita said, I have ₹ 5.75 and Shama said, “I have ₹ 7.50”.

Have they written correctly?

We know that the dot represents a decimal point.

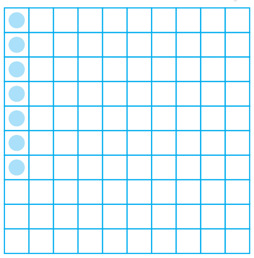
In this chapter, we will learn more about working with decimals.

8.2 Comparing Decimals

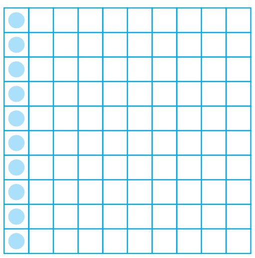
Can you tell which is greater, 0.07 or 0.1?

Take two pieces of square papers of the same size. Divide them into 100 equal parts. For 0.07 we have to shade 7 parts out of 100.

Now, $0.1 = \frac{1}{10} = \frac{10}{100}$, so, for 0.1, shade 10 parts out 100.



$0.07 = \frac{7}{100}$



$0.1 = \frac{1}{10} = \frac{10}{100}$

This means 0.1>0.07

Let us now compare the numbers 32.55 and 32.5. In this case , we first compare the whole part. We see that the whole part for both the nunbers is 32 and, hence, equal.

We, however, know that the two numbers are not equal. So, we now compare the tenth part. We find that for 32.55 and 32.5, the tenth part is also equal, then we compare the hundredth part.

We find,

$32.55 = 32 + \frac{5}{10} + \frac{5}{100}$ and $32.5 = 32 + \frac{5}{10} + \frac{0}{100}$, therefore, 32.55>32.5 as the hundredth part of 32.55 is more.

Example 1 : Which is greater?

(a) 1 or 0.99 (b) 1.09 or 1.093

Solution : (a) $1 = 1 + \frac{0}{10} + \frac{0}{100}$; $0.99 = 0 + \frac{9}{10} + \frac{9}{100}$

The whole part of 1 is greater than that of 0.99.

Therefore, 1 > 0.99

(b) $1.09 = 1 + \frac{0}{10} + \frac{9}{100} + \frac{0}{1000}$; $1.093 = 1 + \frac{0}{10} + \frac{9}{100} + \frac{3}{1000}$

In this case, the two numbers have same parts upto hundredth.

But the thousandths part of 1.093 is greater than that of 1.09.

Therefore, 1.093 > 1.09.

EXERCISE 8.1

1. Which is greater?

(a) 0.3 or 0.4 (b) 0.07 or 0.02 (c) 3 or 0.8 (d) 0.5 or 0.05

(e) 1.23 or 1.2 (f) 0.099 or 0.19 (g) 1.5 or 1.50 (h) 1.431 or 1.490

(i) 3.3 or 3.300 (j) 5.64 or 5.603

2. Make five more examples and find the greater number from them.

Try These

(i) Write 2 rupees 5 paise and 2 rupees 50 paise in decimals.

(ii) Write 20 rupees 7 paise and 21 rupees 75 paise in decimals?

8.3 Using Decimals

8.3.1 Money

We know that 100 paise = ₹ 1

Therefore, 1 paise $= ₹ \frac{1}{100} = ₹ 0.01$

So, 65 paise $= ₹\ \frac{65}{100} = ₹\ 0.65$

and 5 paise $= ₹\ \frac{5}{100} = ₹\ 0.05$

What is 105 paise? It is ₹ 1 and 5 paise = ₹ 1.05

8.3.2 Length

Mahesh wanted to measure the length of his table top in metres. He had a 50 cm scale. He found that the length of the table top was 156 cm. What will be its length in metres?

Mahesh knew that

$1\text{ cm} = \frac{1}{100}\text{ m}$ or 0.01 m

Therefore, $56\text{ cm} = \frac{56}{100}\text{ m} = 0.56\text{ m}$

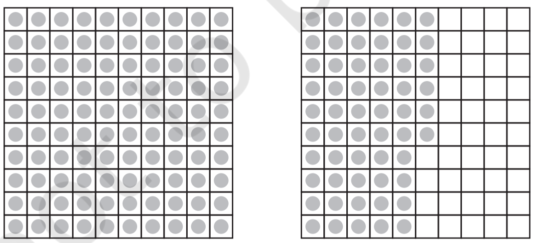
Thus, the length of the table top is
156 cm = 100 cm + 56 cm

$= 1\text{ m} + \frac{56}{100}\text{ m} = 1.56\text{ m}.$

Try These

1. Can you write 4 mm in 'cm' using decimals?
2. How will you write 7cm 5 mm in 'cm' using decimals?
3. Can you now write 52 m as 'km' using decimals? How will you write 340 m as 'km' using decimals? How will you write 2008 m in 'km'?

Mahesh also wants to represent this length pictorially. He took squared papers of equal size and divided them into 100 equal parts. He considered each small square as one cm.



100 cm   56 cm

8.3.3 Weight

Nandu bought 500g potatoes, 250g capsicum, 700g onions, 500g tomatoes, 100g ginger and 300g radish. What is the total weight of the vegetables in the bag? Let us add the weight of all the vegetables in the bag.

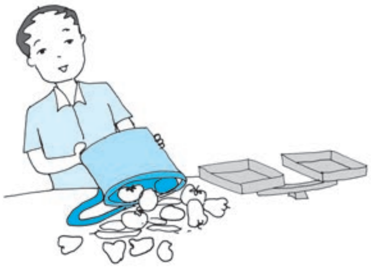
500 g + 250 g + 700 g + 500 g + 100 g + 300 g
= 2350 g

Try These

1. Can you now write 456g as 'kg' using decimals?
2. How will you write 2kg 9g in 'kg' using decimals?

We know that 1000 g = 1 kg

Therefore, 1 g $= \frac{1}{1000}$ kg = 0.001 kg



$$\text{Thus, } 2350 \text{ g} = 2000 \text{ g} + 350 \text{ g}$$
$$= \frac{2000}{1000} \text{ kg} + \frac{350}{1000} \text{ kg}$$
$$= 2 \text{ kg} + 0.350 \text{ kg} = 2.350 \text{ kg}$$
$$\text{i.e. } 2350 \text{ g} = 2 \text{ kg } 350 \text{ g} = 2.350 \text{ kg}$$

Thus, the weight of vegetables in Nandu's bag is 2.350 kg.

EXERCISE 8.2

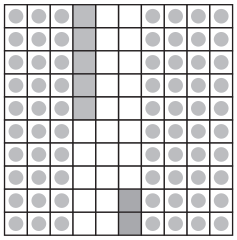
1. Express as rupees using decimals.
   (a) 5 paise (b) 75 paise (c) 20 paise
   (d) 50 rupees 90 paise (e) 725 paise
2. Express as metres using decimals.
   (a) 15 cm (b) 6 cm (c) 2 m 45 cm
   (d) 9 m 7 cm (e) 419 cm
3. Express as cm using decimals.
   (a) 5 mm (b) 60 mm (c) 164 mm
   (d) 9 cm 8 mm (e) 93 mm
4. Express as km using decimals.
   (a) 8 m (b) 88 m (c) 8888 m
   (d) 70 km 5 m
5. Express as kg using decimals.
   (a) 2 g (b) 100 g (c) 3750 g
   (d) 5 kg 8 g (e) 26 kg 50 g

8.4 Addition of Numbers with Decimals

Do This

Add 0.35 and 0.42.

Take a square and divide it into 100 equal parts.



Mark 0.35 in this square by shading 3 tenths and colouring 5 hundredths.

Mark 0.42 in this square by shading 4 tenths and colouring 2 hundredths.

Now count the total number of tenths in the square and the total number of hundredths in the square.

	Ones	Tenths	Hundredths
	0	3	5
+	0	4	2
	0	7	7

Therefore, 0.35 + 0.42 = 0.77

Thus, we can add decimals in the same way as whole numbers.

Can you now add 0.68 and 0.54?

Try These

Find

(i) 0.29 + 0.36 (ii) 0.7 + 0.08

(iii) 1.54 + 1.80 (iv) 2.66 + 1.85

	Ones	Tenths	Hundredths
	0	6	8
+	0	5	4
	1	2	2

Thus, 0.68 + 0.54 = 1.22

Example 2 : Lata spent ₹ 9.50 for buying a pen and ₹ 2.50 for one pencil. How much money did she spend?

Solution : Money spent for pen = ₹ 9.50

Money spent for pencil = ₹ 2.50

Total money spent = ₹ 9.50 + ₹ 2.50

Total money spent = ₹ 12.00

Example 3 : Samson travelled 5 km 52 m by bus, 2 km 265 m by car and the rest 1km 30 m he walked. How much distance did he travel in all?

Solution: Distance travelled by bus = 5 km 52 m = 5.052 km

Distance travelled by car = 2 km 265 m = 2.265 km

Distance travelled on foot = 1 km 30 m = 1.030 km

Therefore, total distance travelled is

$$\begin{array}{r} 5.052 \text{ km} \\ 2.265 \text{ km} \\ + \quad 1.030 \text{ km} \\ \hline 8.347 \text{ km} \\ \hline \end{array}$$

Therefore, total distance travelled = 8.347 km

Example 4 : Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. Find the total weight of all the fruits he bought.

Solution : Weight of apples = 4 kg 90 g = 4.090 kg

Weight of grapes = 2 kg 60 g = 2.060 kg

Weight of mangoes = 5 kg 300 g = 5.300 kg

Therefore, the total weight of the fruits bought is

$$\begin{array}{r} 4.090 \text{ kg} \\ 2.060 \text{ kg} \\ + \quad 5.300 \text{ kg} \\ \hline 11.450 \text{ kg} \\ \hline \end{array}$$

Total weight of the fruits bought = 11.450 kg.

EXERCISE 8.3

1. Find the sum in each of the following :
   (a) 0.007 + 8.5 + 30.08
   (b) 15 + 0.632 + 13.8
   (c) 27.076 + 0.55 + 0.004
   (d) 25.65 + 9.005 + 3.7
   (e) 0.75 + 10.425 + 2
   (f) 280.69 + 25.2 + 38
2. Rashid spent ₹ 35.75 for Maths book and ₹ 32.60 for Science book. Find the total amount spent by Rashid.
3. Radhika's mother gave her ₹ 10.50 and her father gave her ₹ 15.80, find the total amount given to Radhika by the parents.
4. Nasreen bought 3 m 20 cm cloth for her shirt and 2 m 5 cm cloth for her trouser. Find the total length of cloth bought by her.
5. Naresh walked 2 km 35 m in the morning and 1 km 7 m in the evening. How much distance did he walk in all?

6. Sunita travelled 15 km 268 m by bus, 7 km 7 m by car and 500 m on foot in order to reach her school. How far is her school from her residence?
7. Ravi purchased 5 kg 400 g rice, 2 kg 20 g sugar and 10 kg 850g flour. Find the total weight of his purchases.

8.5 Subtraction of Decimals

Do This

Subtract 1.32 from 2.58

This can be shown by the table.

	Ones	Tenths	Hundredths
	2	5	8
–	1	3	2
	1	2	6

Thus, 2.58 – 1.32 = 1.26

Therefore, we can say that, subtraction of decimals can be done by subtracting hundredths from hundredths, tenths from tenths, ones from ones and so on, just as we did in addition.

Sometimes while subtracting decimals, we may need to regroup like we did in addition.

Let us subtract 1.74 from 3.5.

	Ones	Tenths	Hundredths
	3	5	0
–	1	7	4
	1	7	6

Subtract in the hundredth place.

Can't subtract !

so regroup

$$\begin{array}{r} {}^{2}\not{3}\ .\ {}^{14}\not{5}\ {}^{10}0 \\ -\ 1\ .\ 7\ 4 \\ \hline 1\ .\ 7\ 6 \\ \hline \end{array}$$

Thus, 3.5 – 1.74 = 1.76

Try These

1. Subtract 1.85 from 5.46 ;
2. Subtract 5.25 from 8.28 ;
3. Subtract 0.95 from 2.29 ;
4. Subtract 2.25 from 5.68.

Example 5 : Abhishek had ₹ 7.45. He bought toffees for ₹ 5.30. Find the balance amount left with Abhishek.

Solution : Total amount of money = ₹ 7.45

Amount spent on toffees = ₹ 5.30

Balance amount of money = ₹ 7.45 – ₹ 5.30 = ₹ 2.15

Example 6 : Urmila's school is at a distance of 5 km 350 m from her house. She travels 1 km 70 m on foot and the rest by bus. How much distance does she travel by bus?

Solution : Total distance of school from the house = 5.350 km

Distance travelled on foot = 1.070 km

Therefore, distance travelled by bus = 5.350 km – 1.070 km

= 4.280 km

Thus, distance travelled by bus = 4.280 km or 4 km 280 m

Example 7 : Kanchan bought a watermelon weighing 5 kg 200 g. Out of this she gave 2 kg 750 g to her neighbour. What is the weight of the watermelon left with Kanchan?

Solution : Total weight of the watermelon = 5.200 kg

Watermelon given to the neighbour = 2.750 kg

Therefore, weight of the remaining watermelon

= 5.200 kg – 2.750 kg = 2.450 kg

EXERCISE 8.4

1. Subtract :
   (a) ₹ 18.25 from ₹ 20.75
   (b) 202.54 m from 250 m
   (c) ₹ 5.36 from ₹ 8.40
   (d) 2.051 km from 5.206 km
   (e) 0.314 kg from 2.107 kg

2. Find the value of :
   (a) 9.756 – 6.28
   (b) 21.05 – 15.27
   (c) 18.5 – 6.79
   (d) 11.6 – 9.847

3. Raju bought a book for ₹ 35.65. He gave ₹ 50 to the shopkeeper. How much money did he get back from the shopkeeper?
4. Rani had ₹ 18.50. She bought one ice-cream for ₹ 11.75. How much money does she have now?

5. Tina had 20 m 5 cm long cloth. She cuts 4 m 50 cm length of cloth from this for making a curtain. How much cloth is left with her?

6. Namita travels 20 km 50 m every day. Out of this she travels 10 km 200 m by bus and the rest by auto. How much distance does she travel by auto?
7. Aakash bought vegetables weighing 10 kg. Out of this, 3 kg 500 g is onions, 2 kg 75 g is tomatoes and the rest is potatoes. What is the weight of the potatoes?

What have we discussed?

1. Every decimal can be written as a fraction.
2. Any two decimal numbers can be compared among themselves. The comparison can start with the whole part. If the whole parts are equal then the tenth parts can be compared and so on.
3. Decimals are used in many ways in our lives. For example, in representing units of money, length and weight.

Data Handling

Chapter 9

9.1 Introduction

You must have observed your teacher recording the attendance of students in your class everyday, or recording marks obtained by you after every test or examination. Similarly, you must have also seen a cricket score board. Two score boards have been illustrated here :

Name of the bowlers	Overs	Maiden overs	Runs given	Wickets taken
A	10	2	40	3
B	10	1	30	2
C	10	2	20	1
D	10	1	50	4

Name of the batsmen	Runs	Balls faced	Time (in min.)
E	45	62	75
F	55	70	81
G	37	53	67
H	22	41	55

You know that in a game of cricket the information recorded is not simply about who won and who lost. In the score board, you will also find some equally important information about the game. For instance, you may find out the time taken and number of balls faced by the highest run-scorer.

Similarly, in your day to day life, you must have seen several kinds of tables consisting of numbers, figures, names etc.

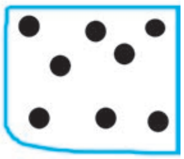
These tables provide 'Data'. *A data is a collection of numbers gathered to give some information.*

9.2 Recording Data

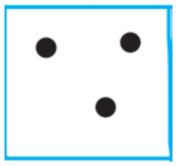
Let us take an example of a class which is preparing to go for a picnic. The teacher asked the students to give their choice of fruits out of banana, apple, orange or guava. Uma is asked to prepare the list. She prepared a list of all the children and wrote the choice of fruit against each name. This list would help the teacher to distribute fruits according to the choice.

Raghav	—	Banana	Bhawana	—	Apple
Preeti	—	Apple	Manoj	—	Banana
Amar	—	Guava	Donald	—	Apple
Fatima	—	Orange	Maria	—	Banana
Amita	—	Apple	Uma	—	Orange
Raman	—	Banana	Akhtar	—	Guava
Radha	—	Orange	Ritu	—	Apple
Farida	—	Guava	Salma	—	Banana
Anuradha	—	Banana	Kavita	—	Guava
Rati	—	Banana	Javed	—	Banana

If the teacher wants to know the number of bananas required for the class, she has to read the names in the list one by one and count the total number of bananas required. To know the number of apples, guavas and oranges separately she has to repeat the same process for each of these fruits. How tedious and time consuming it is! It might become more tedious if the list has, say, 50 students.



Banana



Orange

Apple

So, Uma writes only the names of these fruits one by one like, banana, apple, guava, orange, apple, banana, orange, guava, banana, banana, apple, banana, apple, banana, orange, guava, apple, banana, guava, banana.

Do you think this makes the teacher's work easier? She still has to count the fruits in the list one by one as she did earlier.

Salma has another idea. She makes four squares on the floor. Every square is kept for fruit of one kind only. She asks the students to put one pebble in the square which matches their choices. i.e. a student opting for banana will put a pebble in the square marked for banana and so on.

Guava

By counting the pebbles in each square, Salma can quickly tell the number of each kind of fruit required. She can get the required information quickly by systematically placing the pebbles in different squares.

Try to perform this activity for 40 students and with names of any four fruits. Instead of pebbles you can also use bottle caps or some other tokens.

9.3 Organisation of Data

To get the same information which Salma got, Ronald needs only a pen and a paper. He does not need pebbles. He also does not ask students to come and place the pebbles. He prepares the following table.

Banana	✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓	8
Orange	✓ ✓ ✓	3
Apple	✓ ✓ ✓ ✓ ✓	5
Guava	✓ ✓ ✓ ✓	4

Do you understand Ronald's table?

What does one (✓) mark indicate?

Four students preferred guava. How many (✓) marks are there against guava?

How many students were there in the class? Find all this information.

Discuss about these methods. Which is the best? Why? Which method is more useful when information from a much larger data is required?

Example 1 : A teacher wants to know the choice of food of each student as part of the mid-day meal programme. The teacher assigns the task of collecting this information to Maria. Maria does so using a paper and a pencil. After arranging the choices in a column, she puts against a choice of food one (|) mark for every student making that choice.

<table>
<tr><th>Choice</th><th>Number of students</th></tr>
<tr><td>Rice only</td><td>| | | | | | | | | | | | | | | | |</td></tr>
<tr><td>Chapati only</td><td>| | | | | | | | | | | | |</td></tr>
<tr><td>Both rice and chapati</td><td>| | | | | | | | | | | | | | | | | | | |</td></tr>
</table>

Umesh, after seeing the table suggested a better method to count the students. He asked Maria to organise the marks (|) in a group of ten as shown below :

Choice	Tally marks	Number of students
Rice only	(||||||||||) |||||||	17
Chapati only	(||||||||||) |||	13
Both rice and chapati	(||||||||||) (||||||||||)	20

Rajan made it simpler by asking her to make groups of five instead of ten, as shown below :

Choice	Tally marks	Number of students
Rice only	(|||||) (|||||) (|||||) ||	17
Chapati only	(|||||) (|||||) |||	13
Both rice and chapati	(|||||) (|||||) (|||||) (|||||)	20

Teacher suggested that the fifth mark in a group of five marks should be used as a cross, as shown by '𝍸 '. These are **tally marks**. Thus, 𝍸 || shows the count to be five plus two (i.e. seven) and 𝍸 𝍸 shows five plus five (i.e. ten).

With this, the table looks like :

Choice	Tally marks	Number of students
Rice only	𝍸 𝍸 𝍸 ||	17
Chapati only	𝍸 𝍸 |||	13
Both rice and chapati	𝍸 𝍸 𝍸 𝍸	20

Example 2 : Ekta is asked to collect data for size of shoes of students in her Class VI. Her finding are recorded in the manner shown below :

5	4	7	5	6	7	6	5	6	6	5
4	5	6	8	7	4	6	5	6	4	6
5	7	6	7	5	7	6	4	8	7	

Javed wanted to know (i) the size of shoes worn by the maximum number of students. (ii) the size of shoes worn by the minimum number of students. Can you find this information?

Ekta prepared a table using tally marks.

Shoe size	Tally marks	Number of students
4	𝍸	5
5	𝍸 |||	8
6	𝍸 𝍸	10
7	𝍸 ||	7
8	||	2

Now the questions asked earlier could be answered easily.

You may also do some such activity in your class using tally marks.

Do This

1. Collect information regarding the number of family members of your classmates and represent it in the form of a table. Find to which category most students belong.

Number of family members	Tally marks	Number of students with that many family members

Make a table and enter the data using tally marks. Find the number that appeared

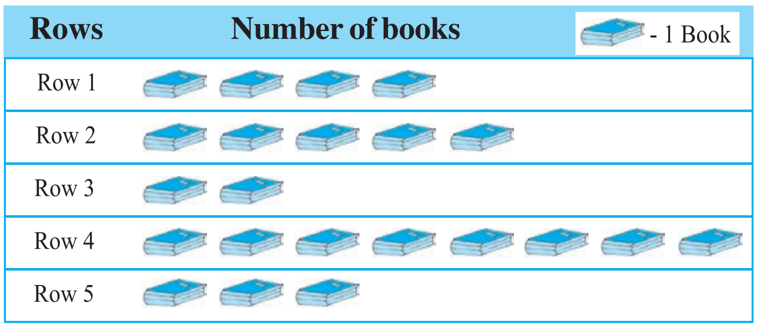
(a) the minimum number of times? (b) the maximum number of times?

(c) same number of times?

9.4 Pictograph

A cupboard has five compartments. In each compartment a row of books is arranged.

The details are indicated in the adjoining table :



Which row has the greatest number of books? Which row has the least number of books? Is there any row which does not have books?

You can answer these questions by just studying the diagram. The picture visually helps you to understand the data. It is a **pictograph**.

A pictograph represents data through pictures of objects. It helps answer the questions on the data at a glance.

Do This

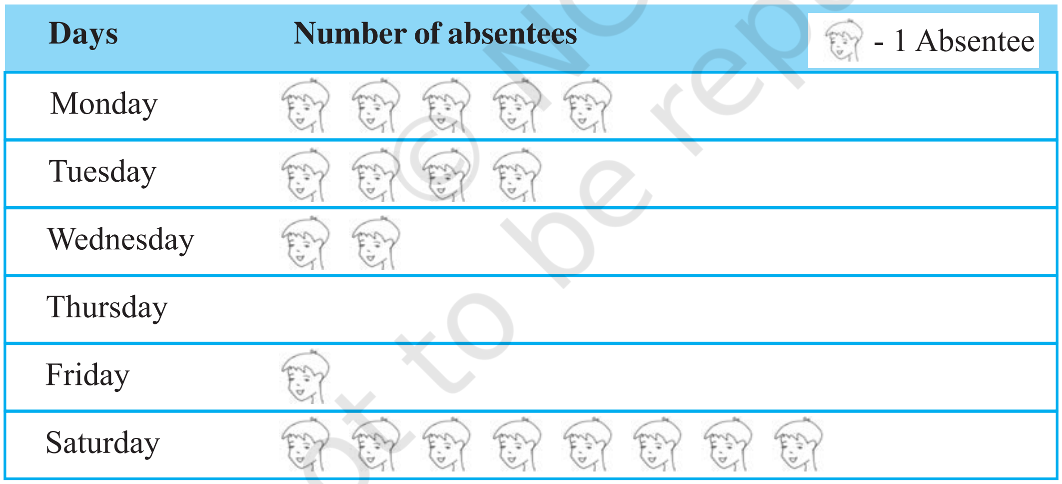
Pictographs are often used by dailies and magazines to attract readers attention.

Collect one or two such published pictographs and display them in your class. Try to understand what they say.

It requires some practice to understand the information given by a pictograph.

9.5 Interpretation of a Pictograph

Example 3 : The following pictograph shows the number of absentees in a class of 30 students during the previous week :



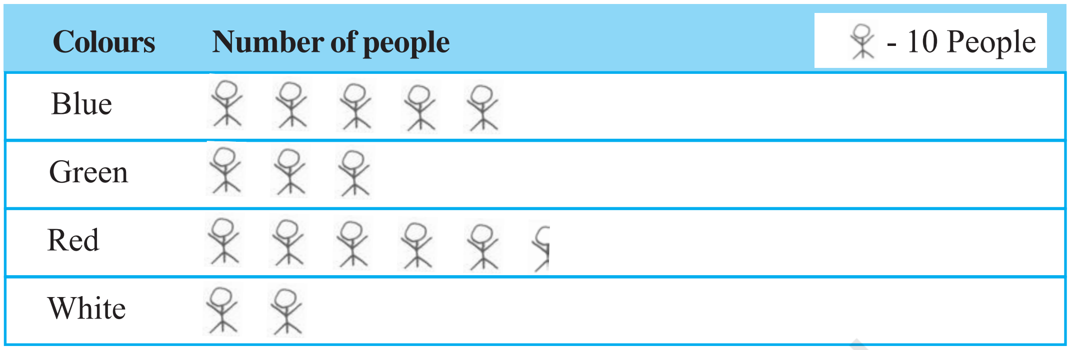
(a) On which day were the maximum number of students absent?
(b) Which day had full attendance?
(c) What was the total number of absentees in that week?

Solution : (a) Maximum absentees were on saturday. (There are 8 pictures in the row for saturday; on all other days, the number of pictures are less).

(b) Against thursday, there is no picture, i.e. no one is absent. Thus, on that day the class had full attendance.

(c) There are 20 pictures in all. So, the total number of absentees in that week was 20.

Example 4 : The colours of fridges preferred by people living in a locality are shown by the following pictograph :



(a) Find the number of people preferring blue colour.

(b) How many people liked red colour?

Solution : (a) Blue colour is preferred by 50 people.

[ = 10, so 5 pictures indicate 5 × 10 people].

(b) Deciding the number of people liking red colour needs more care.

For 5 complete pictures, we get 5 × 10 = 50 people.

For the last incomplete picture, we may roughly take it as 5.

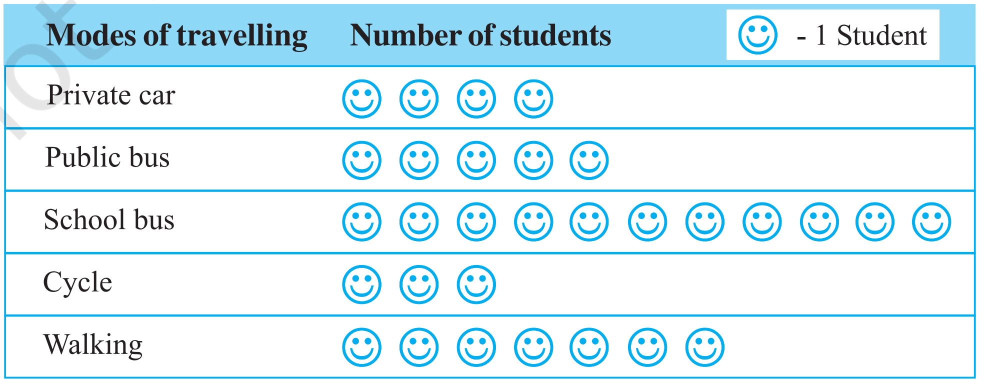
So, number of people preferring red colour is nearly 55.

Think, discuss and write

In the above example, the number of people who like red colour was taken as 50 + 5. If your friend wishes to take it as 50 + 8, is it acceptable?

Example 5 : A survey was carried out on 30 students of class VI in a school. Data about the different modes of transport used by them to travel to school was displayed as pictograph.

What can you conclude from the pictograph?



Solution : From the pictograph we find that:

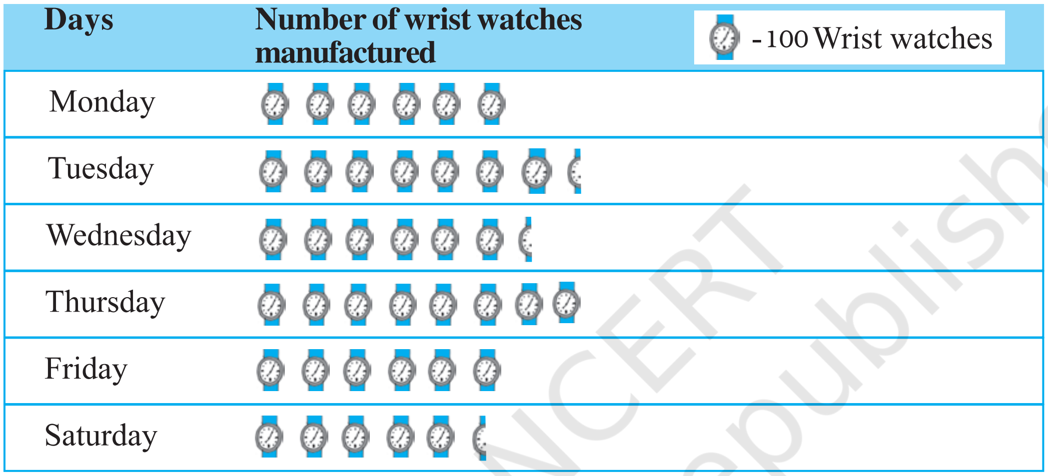
(a) The number of students coming by private car is 4.

(b) Maximum number of students use the school bus. This is the most popular way.

(c) Cycle is used by only three students.

(d) The number of students using the other modes can be similarly found.

Example 6 : Following is the pictograph of the number of wrist watches manufactured by a factory in a particular week.



(a) On which day were the least number of wrist watches manufactured?

(b) On which day were the maximum number of wrist watches manufactured?

(c) Find out the approximate number of wrist watches manufactured in the particular week?

Solution : We can complete the following table and find the answers.

Days	Number of wrist watches manufactured
Monday	600
Tuesday	More than 700 and less than 800
Wednesday	....
Thursday	....
Friday	....
Saturday	....

EXERCISE 9.1

1. In a Mathematics test, the following marks were obtained by 40 students. Arrange these marks in a table using tally marks.

8	1	3	7	6	5	5	4	4	2
4	9	5	3	7	1	6	5	2	7
7	3	8	4	2	8	9	5	8	6
7	4	5	6	9	6	4	4	6	6

(a) Find how many students obtained marks equal to or more than 7.

(b) How many students obtained marks below 4?

2. Following is the choice of sweets of 30 students of Class VI.

Ladoo, Barfi, Ladoo, Jalebi, Ladoo, Rasgulla, Jalebi, Ladoo, Barfi, Rasgulla, Ladoo, Jalebi, Jalebi, Rasgulla, Ladoo, Rasgulla, Jalebi, Ladoo, Rasgulla, Ladoo, Ladoo, Barfi, Rasgulla, Rasgulla, Jalebi, Rasgulla, Ladoo, Rasgulla, Jalebi, Ladoo.

(a) Arrange the names of sweets in a table using tally marks.

(b) Which sweet is preferred by most of the students?

3. Catherine threw a dice 40 times and noted the number appearing each time as shown below :

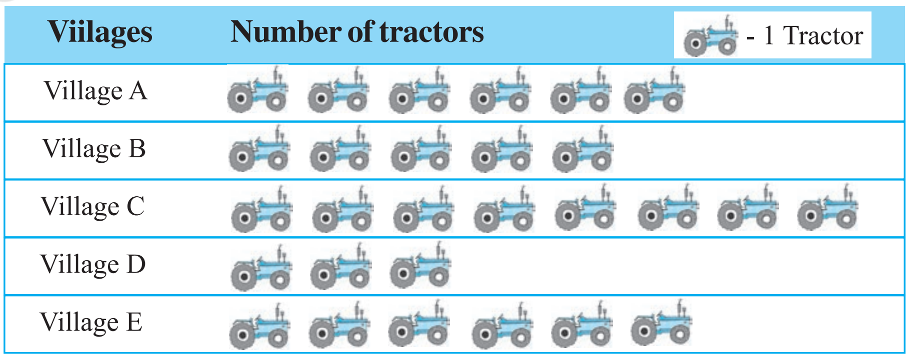
1	3	5	6	6	3	5	4	1	6
2	5	3	4	6	1	5	5	6	1
1	2	2	3	5	2	4	5	5	6
5	1	6	2	3	5	2	4	1	5

Make a table and enter the data using tally marks. Find the number that appeared.

(a) The minimum number of times (b) The maximum number of times

(c) Find those numbers that appear an equal number of times.

4. Following pictograph shows the number of tractors in five villages.



Observe the pictograph and answer the following questions.

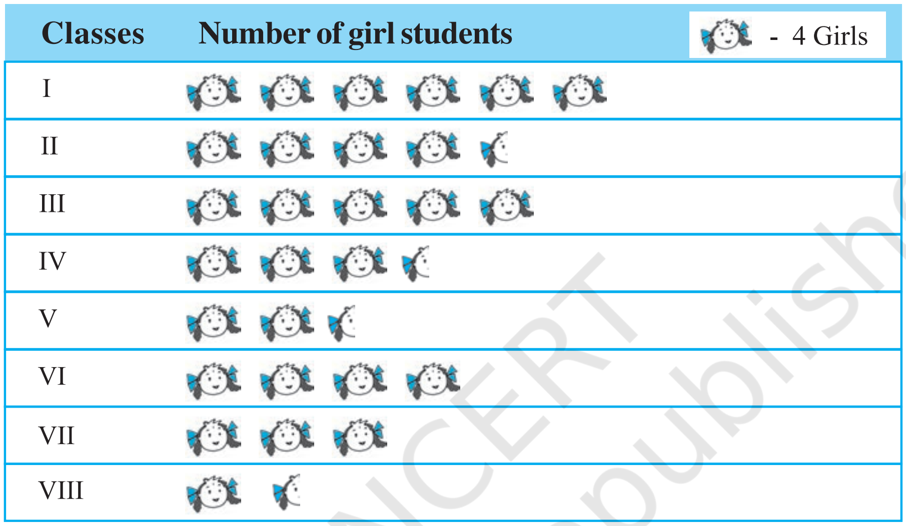
(i) Which village has the minimum number of tractors?

(ii) Which village has the maximum number of tractors?

(iii) How many more tractors village C has as compared to village B.

(iv) What is the total number of tractors in all the five villages?

5. The number of girl students in each class of a co-educational middle school is depicted by the pictograph :



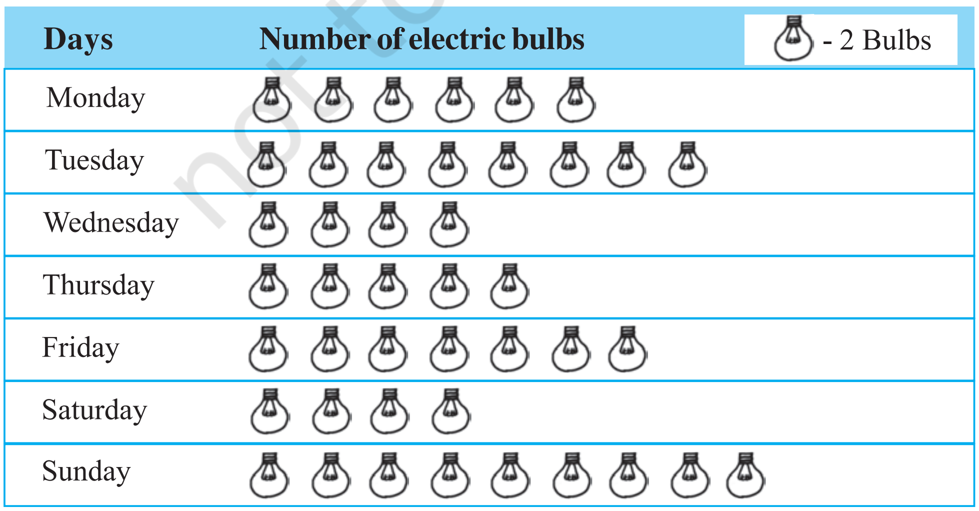
Observe this pictograph and answer the following questions :

(a) Which class has the minimum number of girl students?

(b) Is the number of girls in Class VI less than the number of girls in Class V?

(c) How many girls are there in Class VII?

6. The sale of electric bulbs on different days of a week is shown below :



Observe the pictograph and answer the following questions :

(a) How many bulbs were sold on Friday?

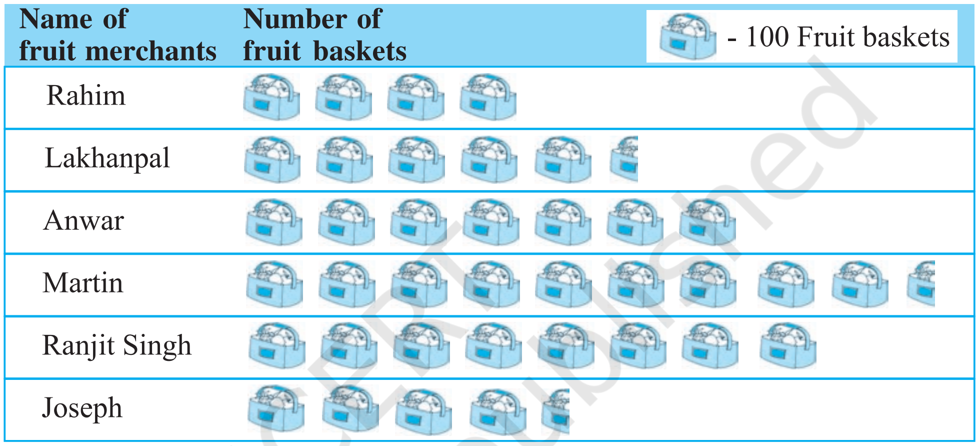
(b) On which day were the maximum number of bulbs sold?

(c) On which of the days same number of bulbs were sold?

(d) On which of the days minimum number of bulbs were sold?

(e) If one big carton can hold 9 bulbs. How many cartons were needed in the given week?

7. In a village six fruit merchants sold the following number of fruit baskets in a particular season :



Observe this pictograph and answer the following questions :

(a) Which merchant sold the maximum number of baskets?

(b) How many fruit baskets were sold by Anwar?

(c) The merchants who have sold 600 or more number of baskets are planning to buy a godown for the next season. Can you name them?

What have we discussed?

1. We have seen that data is a collection of numbers gathered to give some information.

2. To get a particular information from the given data quickly, the data can be arranged in a tabular form using tally marks.

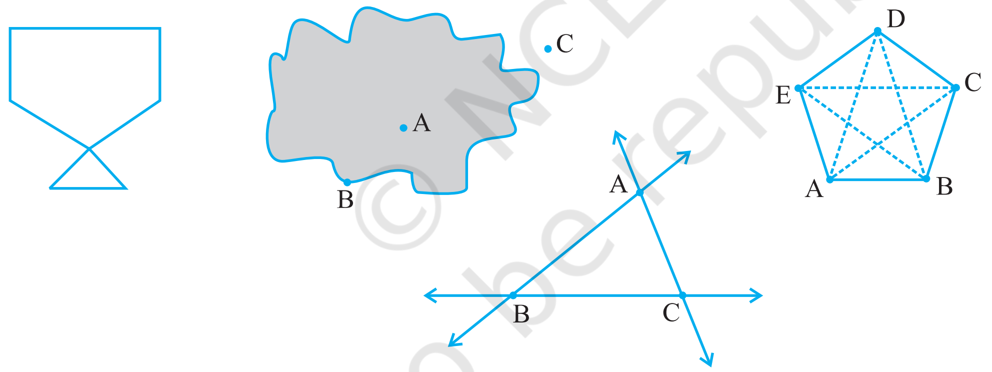
Mensuration

0650CH10

Chapter 10

10.1 Introduction

When we talk about some plane figures as shown below we think of their regions and their boundaries. We need some measures to compare them. We look into these now.



10.2 Perimeter

Look at the following figures (Fig. 10.1). You can make them with a wire or a string.

If you start from the point S in each case and move along the line segments then you again reach the point S. You have made a complete round of the

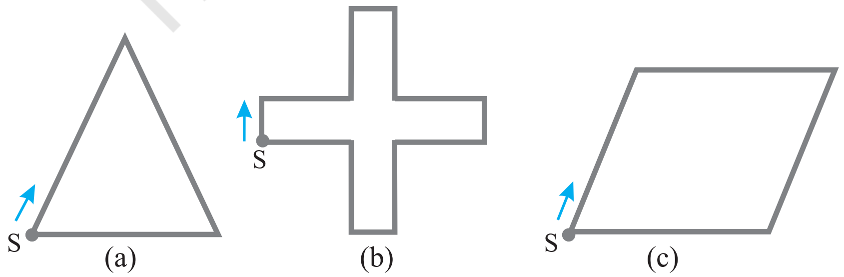


Fig 10.1

shape in each case (a), (b) & (c). The distance covered is equal to the length of wire used to draw the figure.

This distance is known as the **perimeter** of the closed figure. It is the length of the wire needed to form the figures.

The idea of perimeter is widely used in our daily life.

- A farmer who wants to fence his field.
- An engineer who plans to build a compound wall on all sides of a house.
- A person preparing a track to conduct sports.

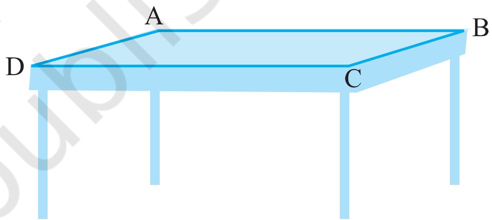
All these people use the idea of 'perimeter'.

Give five examples of situations where you need to know the perimeter.

Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.

Try These

1. Measure and write the length of the four sides of the top of your study table.



AB = ____ cm

BC = ____ cm

CD = ____ cm

DA = ____ cm

Now, the sum of the lengths of the four sides

= AB + BC + CD + DA

= ___ cm +___ cm +___ cm +___ cm

= _____ cm

What is the perimeter?

2. Measure and write the lengths of the four sides of a page of your notebook. The sum of the lengths of the four sides

= AB + BC + CD + DA

= ___ cm +___ cm +___ cm +___ cm

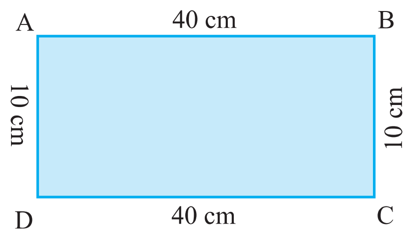
= _____ cm

What is the perimeter of the page?

3. Meera went to a park 150 m long and 80 m wide. She took one complete round on its boundary. What is the distance covered by her?

4. Find the perimeter of the following figures:

(a)

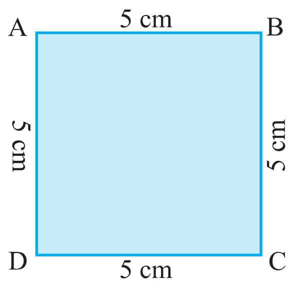


Perimeter = AB + BC + CD + DA

= __+__+__+__

= ______

(b)

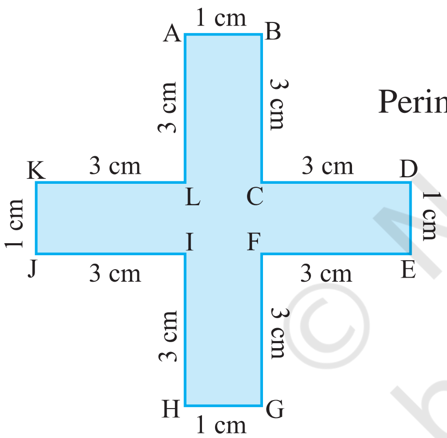


Perimeter = AB + BC + CD + DA

= __ + __ + __+ __

= ______

(c)

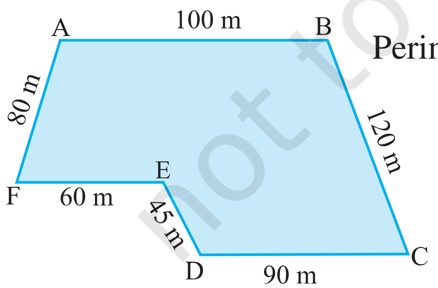


Perimeter = AB + BC + CD + DE + EF + FG + GH +HI + IJ + JK + KL + LA

= __ + __ +__ + __ + __ + __ + __ + __ + __ +__+ __ + __ + __

= ______

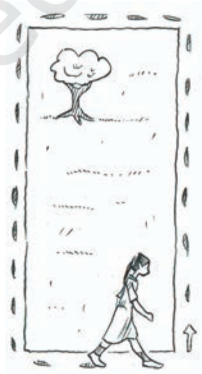
(d)



Perimeter = AB + BC + CD + DE + EF + FA

= __ + __ + __ + __ + __ + __

= ______



So, how will you find the perimeter of any closed figure made up entirely of line segments? Simply find the sum of the lengths of all the sides (which are line segments).

10.2.1 Perimeter of a rectangle

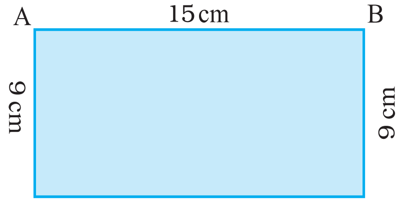
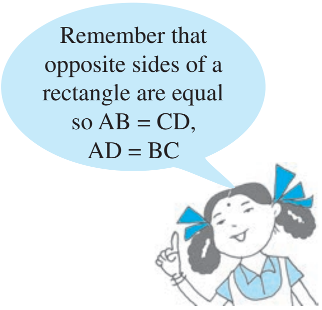


Fig 10.2

Let us consider a rectangle ABCD (Fig 10.2) whose length and breadth are 15 cm and 9 cm respectively. What will be its perimeter?

Perimeter of the rectangle = Sum of the lengths of its four sides.



= AB + BC + CD + DA
= AB + BC + AB + BC
= 2 × AB + 2 × BC
= 2 × (AB + BC)
= 2 × (15cm + 9cm)
= 2 × (24cm)
= 48 cm

Try These

Find the perimeter of the following rectangles:

Length of rectangle	Breadth of rectangle	Perimeter by adding all the sides	Perimeter by 2 × (Length + Breadth)
25 cm	12 cm	= 25 cm + 12 cm + 25 cm + 12 cm = 74 cm	= 2 ×(25 cm + 12 cm) = 2 × (37 cm) = 74 cm
0.5 m	0.25 m		
18 cm	15 cm		
10.5 cm	8.5 cm		

Hence, from the said example, we notice that
Perimeter of a rectangle = length + breadth + length + breadth
i.e. **Perimeter of a rectangle = 2 × (length + breadth)**
Let us now see practical applications of this idea :

Example 1 : Shabana wants to put a lace border all around a rectangular table cover (Fig 10.3), 3 m long and 2 m wide. Find the length of the lace required by Shabana.

Solution : Length of the rectangular table cover = 3 m
Breadth of the rectangular table cover = 2 m

Shabana wants to put a lace border all around the table cover. Therefore, the length of the lace required will be equal to the perimeter of the rectangular table cover.


Fig 10.3

Now, perimeter of the rectangular table cover
= 2 × (length + breadth) = 2 × (3 m + 2 m) = 2 × 5 m = 10 m
So, length of the lace required is 10 m.

Example 2 : An athlete takes 10 rounds of a rectangular park, 50 m long and 25 m wide. Find the total distance covered by him.

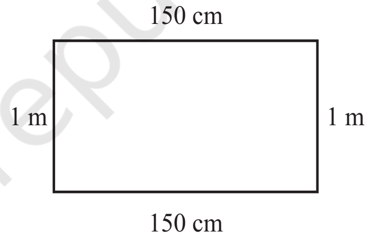
Solution : Length of the rectangular park = 50 m
Breadth of the rectangular park = 25 m

Total distance covered by the athlete in one round will be the perimeter of the park.

Now, perimeter of the rectangular park
= 2 × (length + breadth)= 2 × (50 m + 25 m)
= 2 × 75 m = 150 m
So, the distance covered by the athlete in one round is 150 m.
Therefore, distance covered in 10 rounds = 10 × 150 m = 1500m
The total distance covered by the athlete is 1500 m.

Example 3 : Find the perimeter of a rectangle whose length and breadth are 150 cm and 1 m respectively.

Solution : Length = 150 cm
Breadth = 1m = 100 cm
Perimeter of the rectangle
= 2 × (length + breadth)
= 2 × (150 cm + 100 cm)
= 2 × (250 cm) = 500 cm = 5 m



Example 4 : A farmer has a rectangular field of length and breadth 240 m and 180 m respectively. He wants to fence it with 3 rounds of rope as shown in figure 10.4. What is the total length of rope he must use?

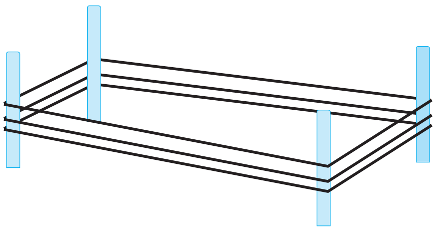


Fig 10.4

Solution : The farmer has to cover three times the perimeter of that field. Therefore, total length of rope required is thrice its perimeter.

Perimeter of the field = 2 × (length + breadth)
= 2 × (240 m + 180 m)
= 2 × 420 m = 840 m

Total length of rope required = 3 × 840 m = 2520 m

Example 5 : Find the cost of fencing a rectangular park of length 250 m and breadth 175 m at the rate of ₹ 12 per metre.

Solution : Length of the rectangular park = 250 m

Breadth of the rectangular park = 175 m

To calculate the cost of fencing we require perimeter.

Perimeter of the rectangle = 2 × (length + breadth)

= 2 × (250 m + 175 m)

= 2 × (425 m) = 850 m

Cost of fencing 1m of park = ₹ 12

Therefore, the total cost of fencing the park

= ₹ 12 × 850 = ₹ 10200

10.2.2 Perimeter of regular shapes

Consider this example.

Biswamitra wants to put coloured tape all around a square picture (Fig 10.5) of side 1m as shown. What will be the length of the coloured tape he requires?

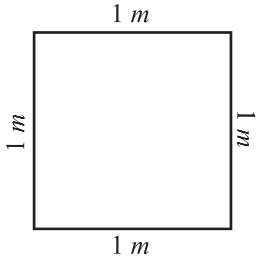


Fig 10.5

Since Biswamitra wants to put the coloured tape all around the square picture, he needs to find the perimeter of the picture frame.

Thus, the length of the tape required

= Perimeter of square = 1m + 1 m + 1 m + 1 m = 4 m

Now, we know that all the four sides of a square are equal, therefore, in place of adding it four times, we can multiply the length of one side by 4. Thus, the length of the tape required = 4 × 1 m = 4 m

From this example, we see that

Perimeter of a square = 4 × length of a side

Draw more such squares and find the perimeters.

Now, look at equilateral triangle (Fig 10.6) with each side equal to 4 cm. Can we find its perimeter?

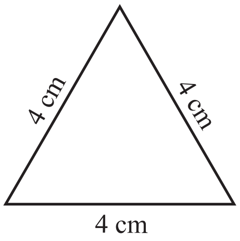


Fig 10.6

Perimeter of this equilateral triangle = 4 + 4 + 4 cm

= 3 × 4 cm = 12 cm

So, we find that

Perimeter of an equilateral triangle = 3 × length of a side

What is similar between a square and an equilateral triangle? They are figures having all the sides of equal length and all the angles of equal measure. Such

Try These

Find various objects from your surroundings which have regular shapes and find their perimeters.

figures are known as *regular closed figures*. Thus, a square and an equilateral triangle are regular closed figures.

You found that,

Perimeter of a square = 4 × length of one side

Perimeter of an equilateral triangle = 3 × length of one side

So, what will be the perimeter of a regular pentagon?

A regular pentagon has five equal sides.

Therefore, perimeter of a regular pentagon = 5 × length of one side and the perimeter of a regular hexagon will be _______ and of an octagon will be _______.

Example 6 : Find the distance travelled by Shaina if she takes three rounds of a square park of side 70 m.

Solution : Perimeter of the square park = 4 × length of a side = 4 × 70 m = 280 m

Distance covered in one round = 280 m

Therefore, distance travelled in three rounds = 3 × 280m = 840m

Example 7 : Pinky runs around a square field of side 75 m, Bob runs around a rectangular field with length 160 m and breadth 105 m. Who covers more distance and by how much?

Solution : Distance covered by Pinky in one round = Perimeter of the square
= 4 × length of a side
= 4 × 75 m = 300 m

Distance covered by Bob in one round = Perimeter of the rectangle
= 2 × (length + breadth)
= 2 × (160 m + 105 m)
= 2 × 265 m = 530 m

Difference in the distance covered = 530 m – 300 m = 230 m.

Therefore, Bob covers more distance by 230 m.

Example 8 : Find the perimeter of a regular pentagon with each side measuring 3 cm.

Solution : This regular closed figure has 5 sides, each with a length of 3 cm. Thus, we get

Perimeter of the regular pentagon = 5 × 3 cm = 15 cm

Example 9 : The perimeter of a regular hexagon is 18 cm. How long is its one side?

Solution : Perimeter = 18 cm

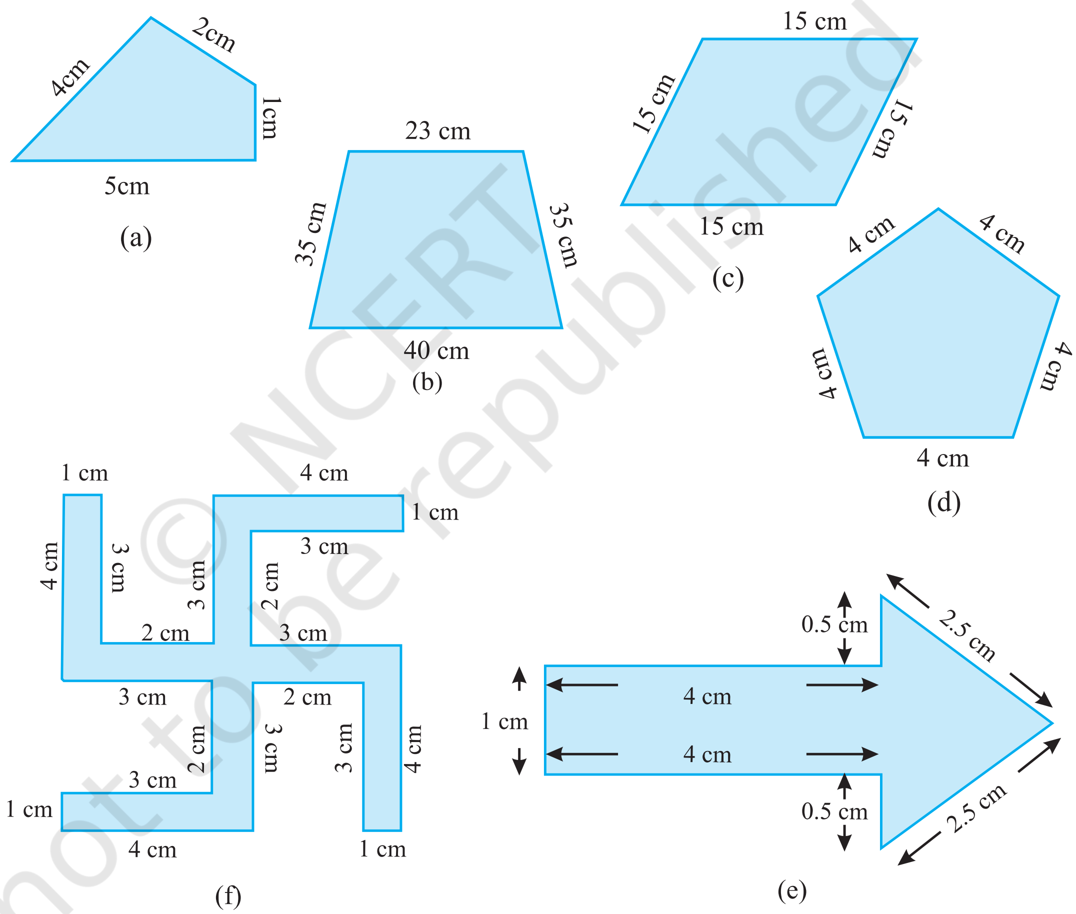
A regular hexagon has 6 sides, so we can divide the perimeter by 6 to get the length of one side.

One side of the hexagon = 18 cm ÷ 6 = 3 cm

Therefore, length of each side of the regular hexagon is 3 cm.

EXERCISE 10.1

1. Find the perimeter of each of the following figures :



2. The lid of a rectangular box of sides 40 cm by 10 cm is sealed all round with tape. What is the length of the tape required?
3. A table-top measures 2 m 25 cm by 1 m 50 cm. What is the perimeter of the table-top?
4. What is the length of the wooden strip required to frame a photograph of length and breadth 32 cm and 21 cm respectively?
5. A rectangular piece of land measures 0.7 km by 0.5 km. Each side is to be fenced with 4 rows of wires. What is the length of the wire needed?

6. Find the perimeter of each of the following shapes :

   (a) A triangle of sides 3 cm, 4 cm and 5 cm.

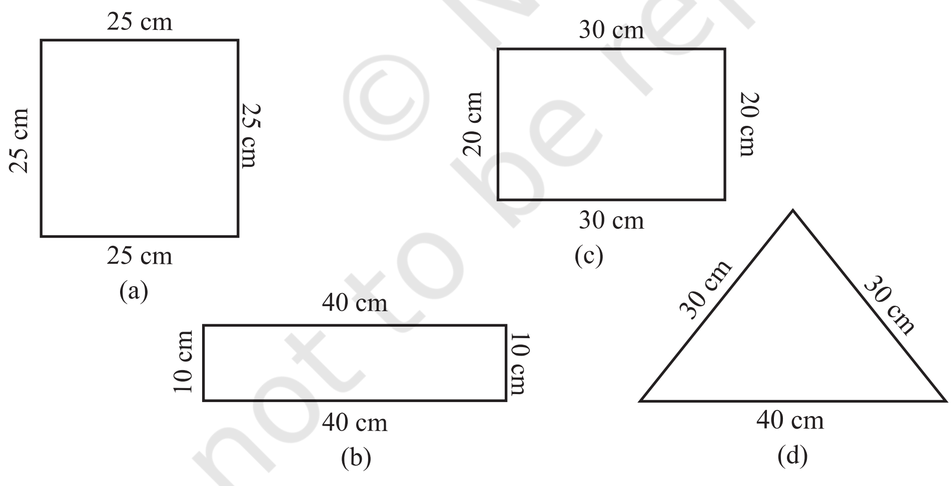
   (b) An equilateral triangle of side 9 cm.

   (c) An isosceles triangle with equal sides 8 cm each and third side 6 cm.

7. Find the perimeter of a triangle with sides measuring 10 cm, 14 cm and 15 cm.
8. Find the perimeter of a regular hexagon with each side measuring 8 m.
9. Find the side of the square whose perimeter is 20 m.
10. The perimeter of a regular pentagon is 100 cm. How long is its each side?
11. A piece of string is 30 cm long. What will be the length of each side if the string is used to form :

    (a) a square? (b) an equilateral triangle? (c) a regular hexagon?

12. Two sides of a triangle are 12 cm and 14 cm. The perimeter of the triangle is 36 cm. What is its third side?
13. Find the cost of fencing a square park of side 250 m at the rate of ₹ 20 per metre.
14. Find the cost of fencing a rectangular park of length 175 m and breadth 125 m at the rate of ₹ 12 per metre.
15. Sweety runs around a square park of side 75 m. Bulbul runs around a rectangular park with length 60 m and breadth 45 m. Who covers less distance?
16. What is the perimeter of each of the following figures? What do you infer from the answers?



17. Avneet buys 9 square paving slabs, each with a side of $\frac{1}{2}$ m. He lays them in the form of a square.

    (a) What is the perimeter of his arrangement [Fig 10.7(i)]?

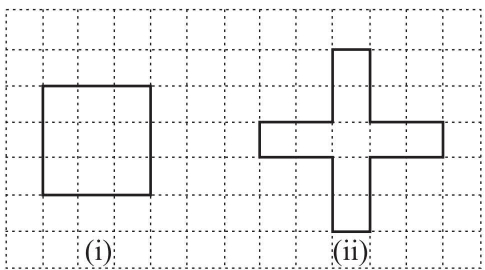


Fig 10.7

(b) Shari does not like his arrangement. She gets him to lay them out like a cross. What is the perimeter of her arrangement [(Fig 10.7 (ii)]?

(c) Which has greater perimeter?

(d) Avneet wonders if there is a way of getting an even greater perimeter. Can you find a way of doing this? (The paving slabs must meet along complete edges i.e. they cannot be broken.)

10.3 Area

Look at the closed figures (Fig 10.8) given below. All of them occupy some region of a flat surface. Can you tell which one occupies more region?

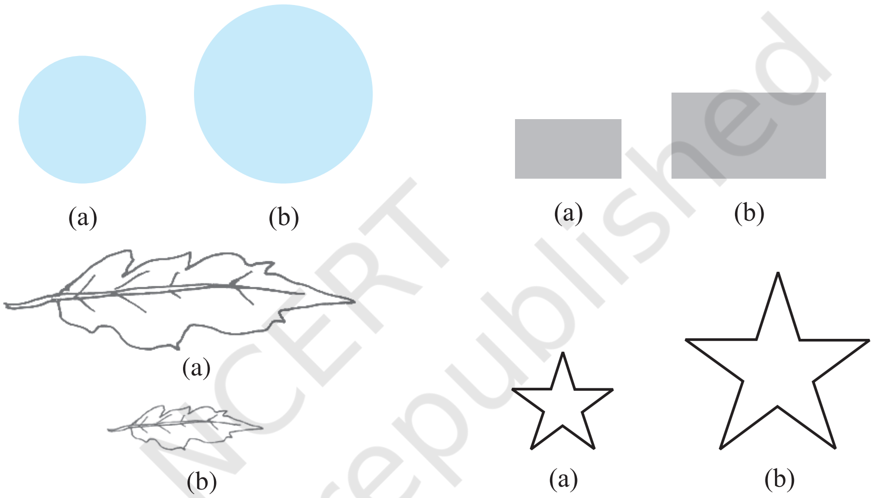


Fig 10.8

The amount of surface enclosed by a closed figure is called its **area.**

So, can you tell, which of the above figures has more area?

Now, look at the adjoining figures of Fig 10.9 :

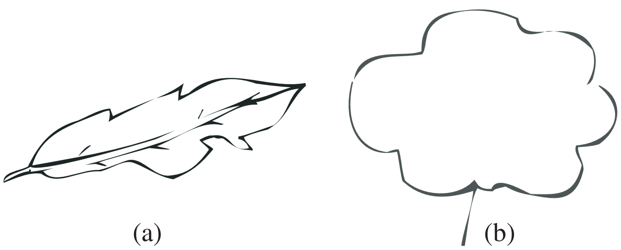


Fig 10.9

Which one of these has larger area? It is difficult to tell just by looking at these figures. So, what do you do?

Place them on a squared paper or graph paper where every square measures 1 cm × 1 cm.

Make an outline of the figure.

Look at the squares enclosed by the figure. Some of them are completely enclosed, some half, some less than half and some more than half.

The area is the number of centimetre squares that are needed to cover it.

But there is a small problem : the squares do not always fit exactly into the area you measure. We get over this difficulty by adopting a convention :

- The area of one full square is taken as 1 sq unit. If it is a centimetre square sheet, then area of one full square will be 1 sq cm.
- Ignore portions of the area that are less than half a square.
- If more than half of a square is in a region, just count it as one square.
- If exactly half the square is counted, take its area as $\frac{1}{2}$ sq unit.

Such a convention gives a fair estimate of the desired area.

Example 10 : Find the area of the shape shown in the figure 10.10.

Solution : This figure is made up of line-segments. Moreover, it is covered by full squares and half squares only. This makes our job simple.

(i) Fully-filled squares = 3

(ii) Half-filled squares = 3

Area covered by full squares

= 3 × 1 sq units = 3 sq units

Total area = $4\frac{1}{2}$ sq units.

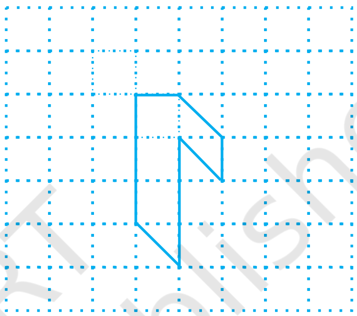


Fig 10.10

Example 11 : By counting squares, estimate the area of the figure 10.9 b.

Soultion : Make an outline of the figure on a graph sheet. (Fig 10.11)

Covered area	Number	Area estimate (sq units)
(i) Fully-filled squares	11	11
(ii) Half-filled squares	3	$3 \times \frac{1}{2}$
(iii) More than half-filled squares	7	7
(iv) Less than half-filled squares	5	0

Total area = $11 + 3 \times \frac{1}{2} + 7 = 19\frac{1}{2}$ sq units.

How do the squares cover it?

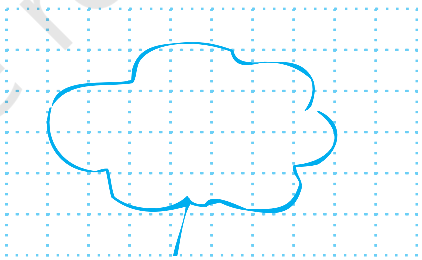


Fig 10.11

Example 12 : By counting squares, estimate the area of the figure 10.9 a.

Soultion : Make an outline of the figure on a graph sheet. This is how the squares cover the figure (Fig 10.12).

Try These

1. Draw any circle on a graph sheet. Count the squares and use them to estimate the area of the circular region.
2. Trace shapes of leaves, flower petals and other such objects on the graph paper and find their areas.

Covered area	Number	Area estimate (sq units)
(i) Fully-filled squares	1	1
(ii) Half-filled squares	–	–
(iii) More than half-filled squares	7	7
(iv) Less than half-filled squares	9	0

Total area = 1 + 7 = 8 sq units.

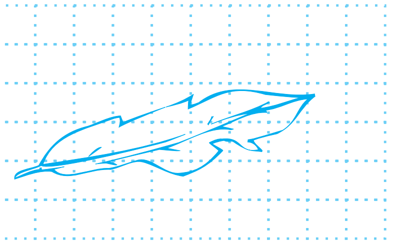
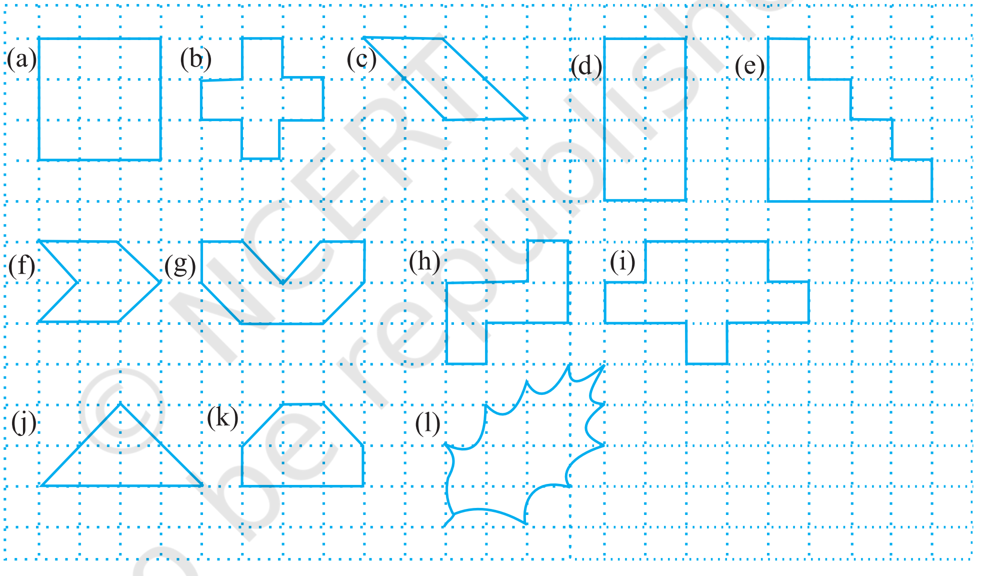
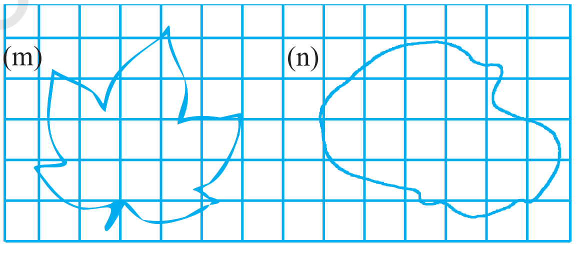


Fig 10.12

EXERCISE 10.2

1. Find the areas of the following figures by counting square:





10.3.1 Area of a rectangle

With the help of the squared paper, can we tell, what will be the area of a rectangle whose length is 5 cm and breadth is 3 cm?

Draw the rectangle on a graph paper having 1 cm × 1 cm squares (Fig 10.13). The rectangle covers 15 squares completely.

The area of the rectangle = 15 sq cm which can be written as 5 × 3 sq cm i.e. (length × breadth).

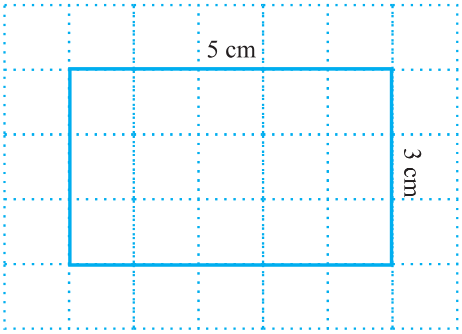


Fig 10.13

The measures of the sides of some of the rectangles are given. Find their areas by placing them on a graph paper and counting the number of square.

Length	Breadth	Area
3 cm	4 cm	----------
7 cm	5 cm	----------
5 cm	3 cm	----------

What do we infer from this?

We find,

Area of a rectangle = (length × breadth)

Without using the graph paper, can we find the area of a rectangle whose length is 6 cm and breadth is 4cm?

Yes, it is possible.

What do we infer from this?

We find that,

Area of the rectangle = length × breadth = 6 cm × 4 cm = 24 sq cm.

Try These

1. Find the area of the floor of your classroom.
2. Find the area of any one door in your house.

10.3.2 Area of a square

Let us now consider a square of side 4 cm (Fig 10.14).

What will be its area?

If we place it on a centimetre graph paper, then what do we observe?

It covers 16 squares i.e. the area of the square = 16 sq cm = 4 × 4 sq cm

Calculate areas of few squares by assuring length of one side of squares by yourself.

Find their areas using graph papers.

What do we infer from this?

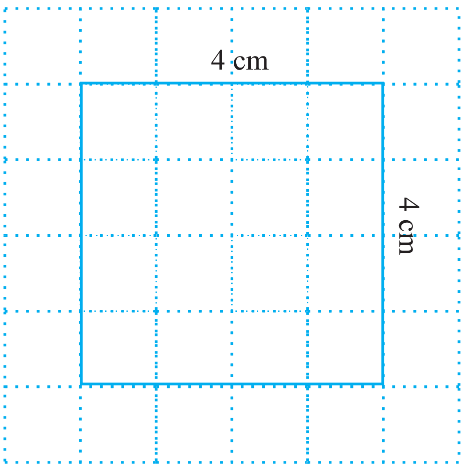


Fig 10.14

We find that in each case,

Area of the square = side × side

You may use this as a formula in doing problems.

Example 13 : Find the area of a rectangle whose length and breadth are 12 cm and 4 cm respectively.

Solution : Length of the rectangle = 12 cm

Breadth of the rectangle = 4 cm

Area of the rectangle = length × breadth

= 12 cm × 4 cm = 48 sq cm.

Example 14 : Find the area of a square plot of side 8 m.

Solution : Side of the square = 8 m

Area of the square = side × side

= 8 m × 8 m = 64 sq m.

Example 15 : The area of a rectangular piece of cardboard is 36 sq cm and its length is 9 cm. What is the width of the cardboard?

Solution : Area of the rectangle = 36 sq cm

Length = 9 cm

Width = ?

Area of a rectangle = length × width

So, width = $\frac{\text{Area}}{\text{Length}} = \frac{36}{9} = 4$ cm

Thus, the width of the rectangular cardboard is 4 cm.

Example 16 : Bob wants to cover the floor of a room 3 m wide and 4 m long by squared tiles. If each square tile is of side 0.5 m, then find the number of tiles required to cover the floor of the room.

Solution : Total area of tiles must be equal to the area of the floor of the room.

Length of the room = 4 m

Breadth of the room = 3 m

Area of the floor = length × breadth

= 4 m × 3 m = 12 sq m

Area of one square tile = side × side

= 0.5 m × 0.5 m

= 0.25 sq m

Number of tiles required = $\dfrac{\text{Area of the floor}}{\text{Area of one tile}} = \dfrac{12}{0.25} = \dfrac{1200}{25} = 48$ tiles.

Example 17 : Find the area in square metre of a piece of cloth 1m 25 cm wide and 2 m long.

Solution : Length of the cloth = 2 m

Breadth of the cloth = 1 m 25 cm = 1 m + 0. 25 m = 1.25 m

(since 25 cm = 0.25m)

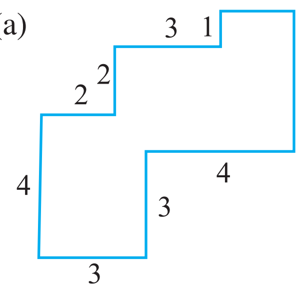
Area of the cloth = length of the cloth × breadth of the cloth

= 2 m × 1.25 m = 2.50 sq m

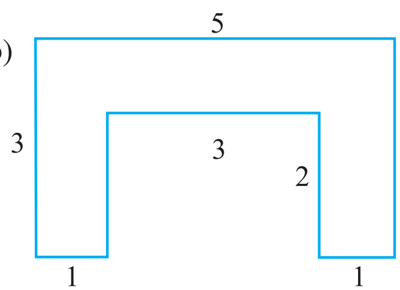
EXERCISE 10.3

1. Find the areas of the rectangles whose sides are :
   (a) 3 cm and 4 cm (b) 12 m and 21 m (c) 2 km and 3 km (d) 2 m and 70 cm
2. Find the areas of the squares whose sides are :
   (a) 10 cm (b) 14 cm (c) 5 m
3. The length and breadth of three rectangles are as given below :
   (a) 9 m and 6 m (b) 17 m and 3 m (c) 4 m and 14 m
   Which one has the largest area and which one has the smallest?
4. The area of a rectangular garden 50 m long is 300 sq m. Find the width of the garden.
5. What is the cost of tiling a rectangular plot of land 500 m long and 200 m wide at the rate of ₹ 8 per hundred sq m.?
6. A table-top measures 2 m by 1 m 50 cm. What is its area in square metres?
7. A room is 4 m long and 3 m 50 cm wide. How many square metres of carpet is needed to cover the floor of the room?
8. A floor is 5 m long and 4 m wide. A square carpet of sides 3 m is laid on the floor. Find the area of the floor that is not carpeted.
9. Five square flower beds each of sides 1 m are dug on a piece of land 5 m long and 4 m wide. What is the area of the remaining part of the land?
10. By splitting the following figures into rectangles, find their areas (The measures are given in centimetres).

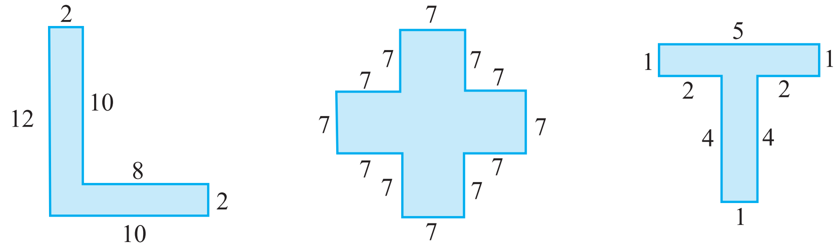
(a)



(b)



11. Split the following shapes into rectangles and find their areas. (The measures are given in centimetres)



(a) (b) (c)

12. How many tiles whose length and breadth are 12 cm and 5 cm respectively will be needed to fit in a rectangular region whose length and breadth are respectively:
(a) 100 cm and 144 cm (b) 70 cm and 36 cm.

A challenge!

On a centimetre squared paper, make as many rectangles as you can, such that the area of the rectangle is 16 sq cm (consider only natural number lengths).

(a) Which rectangle has the greatest perimeter?

(b) Which rectangle has the least perimeter?

If you take a rectangle of area 24 sq cm, what will be your answers?

Given any area, is it possible to predict the shape of the rectangle with the greatest perimeter? With the least perimeter? Give example and reason.

What have we discussed?

1. Perimeter is the distance covered along the boundary forming a closed figure when you go round the figure once.
2. (a) Perimeter of a rectangle = 2 × (length + breadth)

   (b) Perimeter of a square = 4 × length of its side

   (c) Perimeter of an equilateral triangle = 3 × length of a side
3. Figures in which all sides and angles are equal are called regular closed figures.
4. The amount of surface enclosed by a closed figure is called its area.
5. To calculate the area of a figure using a squared paper, the following conventions are adopted :

   (a) Ignore portions of the area that are less than half a square.

   (b) If more than half a square is in a region. Count it as one square.

   (c) If exactly half the square is counted, take its area as $\frac{1}{2}$ sq units.
6. (a) Area of a rectangle = length × breadth

   (b) Area of a square = side × side

Algebra

Chapter 11

0650CH11

11.1 Introduction

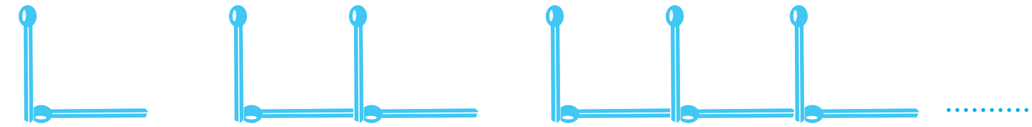
Our study so far has been with numbers and shapes. We have learnt numbers, operations on numbers and properties of numbers. We applied our knowledge of numbers to various problems in our life. The branch of mathematics in which we studied numbers is **arithmetic**. We have also learnt about figures in two and three dimensions and their properties. The branch of mathematics in which we studied shapes is **geometry**. Now we begin the study of another branch of mathematics. It is called **algebra**.

The main feature of the new branch which we are going to study is the use of letters. Use of letters will allow us to write rules and formulas in a general way. By using letters, we can talk about any number and not just a particular number. Secondly, letters may stand for unknown quantities. By learning methods of determining unknowns, we develop powerful tools for solving puzzles and many problems from daily life. Thirdly, since letters stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and their properties.

You will find algebra interesting and useful. It is very useful in solving problems. Let us begin our study with simple examples.

11.2 Matchstick Patterns

Ameena and Sarita are making patterns with matchsticks. They decide to make simple patterns of the letters of the English alphabet. Ameena takes two matchsticks and forms the letter L as shown in Fig 11.1 (a).



(a) (b) (c)

Fig 11.1

Then Sarita also picks two sticks, forms another letter L and puts it next to the one made by Ameena [Fig 11.1 (b)].

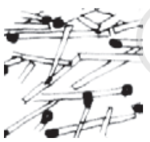
Then Ameena adds one more L and this goes on as shown by the dots in Fig 11.1 (c).

Their friend Appu comes in. He looks at the pattern. Appu always asks questions. He asks the girls, "How many matchsticks will be required to make seven Ls"? Ameena and Sarita are systematic. They go on forming the patterns with 1L, 2Ls, 3Ls, and so on and prepare a table.

Table 1

Number of Ls formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....	....

Appu gets the answer to his question from the Table 1; 7Ls require 14 matchsticks.



While writing the table, Ameena realises that the number of matchsticks required is twice the number of Ls formed.

Number of matchsticks required = 2 × number of Ls.

For convenience, let us write the letter n for the number of Ls. If one L is made, $n = 1$; if two Ls are made, $n = 2$ and so on; thus, n can be any natural number 1, 2, 3, 4, 5, ... We then write, Number of matchsticks required = $2 \times n$.

Instead of writing $2 \times n$, we write $2n$. Note that $2n$ is same as $2 \times n$.

Ameena tells her friends that her rule gives the number of matchsticks required for forming any number of Ls.

Thus, For $n = 1$, the number of matchsticks required $= 2 \times 1 = 2$

For $n = 2$, the number of matchsticks required $= 2 \times 2 = 4$

For $n = 3$, the number of matchsticks required $= 2 \times 3 = 6$ etc.

These numbers agree with those from Table 1.

Sarita says, "The rule is very powerful! Using the rule, I can say how many matchsticks are required to form even 100 Ls. I do not need to draw the pattern or make a table, once the rule is known".
Do you agree with Sarita?

11.3 The Idea of a Variable

In the above example, we found a rule to give the number of matchsticks required to make a pattern of Ls. The rule was :

Number of matchsticks required = 2*n*

Here, *n* is the number of Ls in the pattern, and *n* takes values 1, 2, 3, 4,.... Let us look at Table 1 once again. In the table, the value of *n* goes on changing (increasing). As a result, the number of matchsticks required also goes on changing (increasing).

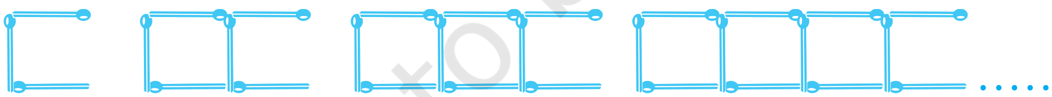
***n* is an example of a variable. Its value is not fixed; it can take any value 1, 2, 3, 4, ... We wrote the rule for the number of matchsticks required using the variable *n*.**

The word 'variable' means something that can vary, i.e. change. The value of a variable is not fixed. It can take different values.

We shall look at another example of matchstick patterns to learn more about variables.

11.4 More Matchstick Patterns

Ameena and Sarita have become quite interested in matchstick patterns. They now want to try a pattern of the letter C. To make one C, they use three matchsticks as shown in Fig. 11.2(a).



(a) (b) (c) (d)

Fig 11.2

Table 2 gives the number of matchsticks required to make a pattern of Cs.

Table 2

Number of Cs formed	1	2	3	4	5	6	7	8	....	....	....
Number of matchsticks required	3	6	9	12	15	18	21	24	....	....	....

Can you complete the entries left blank in the table?

Sarita comes up with the rule :
Number of matchsticks required = 3*n*

She has used the letter *n* for the number of Cs; *n* is a variable taking on values 1, 2, 3, 4, ...

Do you agree with Sarita ?

Remember 3*n* is the same as $3 \times n$.

Next, Ameena and Sarita wish to make a pattern of Fs. They make one F using 4 matchsticks as shown in Fig 11.3(a).

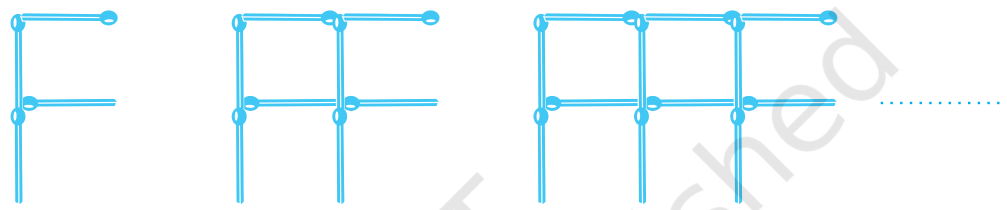


Fig 11.3

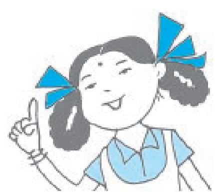
Can you now write the rule for making patterns of F?

Think of other letters of the alphabet and other shapes that can be made from matchsticks. For example, U (⊔), V (\/), triangle (△), square (□) etc. Choose any five and write the rules for making matchstick patterns with them.

11.5 More Examples of Variables

We have used the letter *n* to show a variable. Raju asks, "Why not *m*"? There is nothing special about *n,* any letter can be used.

One may use any letter as *m, l, p, x, y, z* etc. to show a variable. Remember, a variable is a number which does not have a fixed value. For example, the number 5 or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of angles of a triangle has a fixed value i.e. 3. It is not a variable. The number of corners of a quadrilateral (4) is fixed; it is also not a variable. But *n* in the examples we have looked is a variable. It takes on various values 1, 2, 3, 4, ...



Let us now consider variables in a more familiar situation.

Students went to buy notebooks from the school bookstore. Price of one notebook is ₹ 5. Munnu wants to buy 5 notebooks, Appu wants to buy 7 notebooks, Sara wants to buy 4 notebooks and so on. How much money should a student carry when she or he goes to the bookstore to buy notebooks?

This will depend on how many notebooks the student wants to buy. The students work together to prepare a table.

Table 3

Number of notebooks required	1	2	3	4	5	.....	m	.....
Total cost in rupees	5	10	15	20	25	.....	$5m$	.....

The letter m stands for the number of notebooks a student wants to buy; m is a variable, which can take any value 1, 2, 3, 4, ... The total cost of m notebooks is given by the rule :

The total cost in rupees $= 5 \times$ number of note books required

$= 5m$

If Munnu wants to buy 5 notebooks, then taking $m = 5$, we say that Munnu should carry ₹ 5×5 or ₹ 25 with him to the school bookstore.

Let us take one more example. For the Republic Day celebration in the school, children are going to perform mass drill in the presence of the chief guest. They stand 10 in a row (Fig 11.4). How many children can there be in the drill?

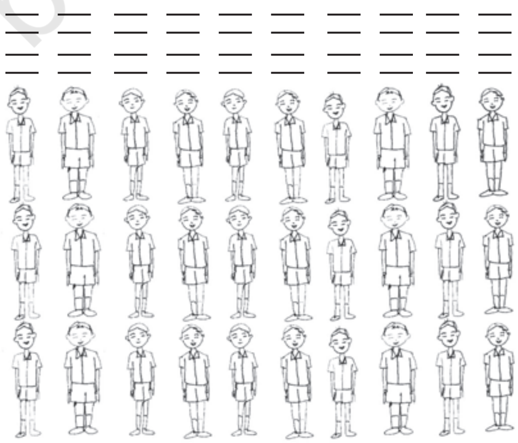


Fig 11.4

The number of children will depend on the number of rows. If there is 1 row, there will be 10 children. If there are 2 rows, there will be 2×10 or 20 children and so on. If there are r rows, there will be $10r$ children in the drill; here, *r* is a variable which stands for the number of rows and so takes on values 1, 2, 3, 4, ...

In all the examples seen so far, the variable was multiplied by a number. There can be different situations as well in which numbers are added to or subtracted from the variable as seen below.

Sarita says that she has 10 more marbles in her collection than Ameena. If Ameena has 20 marbles, then Sarita has 30. If Ameena has 30 marbles, then Sarita has 40 and so on. We do not know exactly how many marbles Ameena has. She may have any number of marbles.

But we know that, Sarita's marbles = Ameena's marbles + 10.

We shall denote Ameena's marbles by the letter x. Here, x is a variable, which can take any value 1, 2, 3, 4,... ,10,... ,20,... ,30,... . Using x, we write Sarita's marbles = $x + 10$. The expression $(x + 10)$ is read as 'x plus ten'. It means 10 added to x. If x is 20, $(x + 10)$ is 30. If x is 30, $(x + 10)$ is 40 and so on.

The expression $(x + 10)$ cannot be simplified further.
Do not confuse $x + 10$ with $10x$, they are different.
In $10x$, x is multiplied by 10. In $(x + 10)$, 10 is added to x.
We may check this for some values of x.
For example,
If $x = 2$, $10x = 10 \times 2 = 20$ and $x + 10 = 2 + 10 = 12$.
If $x = 10$, $10x = 10 \times 10 = 100$ and $x + 10 = 10 + 10 = 20$.

Raju and Balu are brothers. Balu is younger than Raju by 3 years. When Raju is 12 years old, Balu is 9 years old. When Raju is 15 years old, Balu is 12 years old. We do not know Raju's age exactly. It may have any value. Let x denote Raju's age in years, x is a variable. If Raju's age in years is x, then Balu's age in years is $(x – 3)$. The expression $(x – 3)$ is read as x minus three. As you would expect, when x is 12, $(x – 3)$ is 9 and when x is 15, $(x – 3)$ is 12.

EXERCISE 11.1

1. Find the rule which gives the number of matchsticks required to make the following matchstick patterns. Use a variable to write the rule.
   (a) A pattern of letter T as 
   (b) A pattern of letter Z as 

(c) A pattern of letter U as 

(d) A pattern of letter V as 

(e) A pattern of letter E as 

(f) A pattern of letter S as 

(g) A pattern of letter A as 

2. We already know the rule for the pattern of letters L, C and F. Some of the letters from Q.1 (given above) give us the same rule as that given by L. Which are these? Why does this happen?

3. Cadets are marching in a parade. There are 5 cadets in a row. What is the rule which gives the number of cadets, given the number of rows? (Use n for the number of rows.)

4. If there are 50 mangoes in a box, how will you write the total number of mangoes in terms of the number of boxes? (Use b for the number of boxes.)

5. The teacher distributes 5 pencils per student. Can you tell how many pencils are needed, given the number of students? (Use s for the number of students.)

6. A bird flies 1 kilometer in one minute. Can you express the distance covered by the bird in terms of its flying time in minutes? (Use t for flying time in minutes.)

7. Radha is drawing a dot Rangoli (a beautiful pattern of lines joining dots) with chalk powder. She has 9 dots in a row. How many dots will her Rangoli have for r rows? How many dots are there if there are 8 rows? If there are 10 rows?

Fig 11.5

8. Leela is Radha's younger sister. Leela is 4 years younger than Radha. Can you write Leela's age in terms of Radha's age? Take Radha's age to be x years.

9. Mother has made laddus. She gives some laddus to guests and family members; still 5 laddus remain. If the number of laddus mother gave away is l, how many laddus did she make?

10. Oranges are to be transferred from larger boxes into smaller boxes. When a large box is emptied, the oranges from it fill two smaller boxes and still 10 oranges remain outside. If the number of oranges in a small box are taken to be x, what is the number of oranges in the larger box?

11. (a) Look at the following matchstick pattern of squares (Fig 11.6). The squares are not separate. Two neighbouring squares have a common matchstick. Observe the patterns and find the rule that gives the number of matchsticks

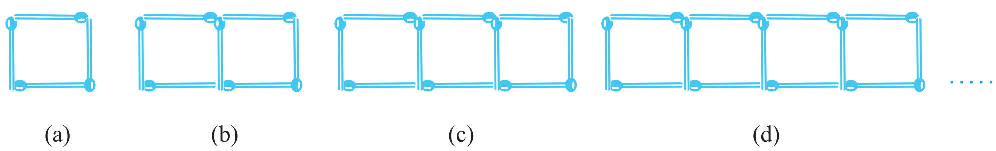
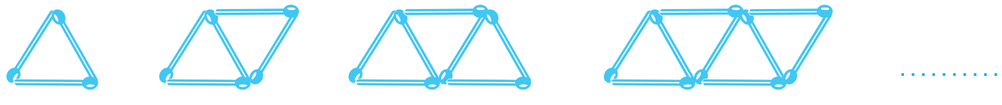


Fig 11.6

in terms of the number of squares. (Hint : If you remove the vertical stick at the end, you will get a pattern of Cs.)

(b) Fig 11.7 gives a matchstick pattern of triangles. As in Exercise 11 (a) above, find the general rule that gives the number of matchsticks in terms of the number of triangles.



(a) (b) (c) (d)

Fig 11.7

What have we discussed?

1. We looked at patterns of making letters and other shapes using matchsticks. We learnt how to write the general relation between the number of matchsticks required for repeating a given shape. The number of times a given shape is repeated varies; it takes on values 1,2,3,... . It is a variable, denoted by some letter like n.
2. A variable takes on different values, its value is not fixed. The length of a square can have any value. It is a variable. But the number of angles of a triangle has a fixed value 3. It is not a variable.
3. We may use any letter n, l, m, p, x, y, z, etc. to show a variable.
4. A variable allows us to express relations in any practical situation.
5. Variables are numbers, although their value is not fixed. We can do the operations of addition, subtraction, multiplication and division on them just as in the case of fixed numbers. Using different operations we can form expressions with variables like $x - 3$, $x + 3$, $2n$, $5m$, $\frac{p}{3}$, $2y + 3$, $3l - 5$, etc.

Ratio and Proportion

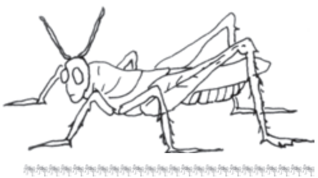
Chapter 12

0650CH12

12.1 Introduction

In our daily life, many a times we compare two quantities of the same type. For example, Avnee and Shari collected flowers for scrap notebook. Avnee collected 30 flowers and Shari collected 45 flowers. So, we may say that Shari collected 45 – 30 = 15 flowers more than Avnee.

Also, if height of Rahim is 150 cm and that of Avnee is 140 cm then, we may say that the height of Rahim is 150 cm – 140 cm = 10 cm more than Avnee. This is one way of comparison by taking difference.



If we wish to compare the lengths of an ant and a grasshopper, taking the difference does not express the comparison. The grasshopper's length, typically 4 cm to 5 cm is too long as compared to the ant's length which is a few mm. Comparison will be better if we try to find that how many ants can be placed one behind the other to match the length of grasshopper. So, we can say that 20 to 30 ants have the same length as a grasshopper.

Consider another example.

Cost of a car is ₹ 2,50,000 and that of a motorbike is ₹ 50,000. If we calculate the difference between the costs, it is ₹ 2,00,000 and if we compare by division;

i.e. $\frac{2,50,000}{50,000} = \frac{5}{1}$

We can say that the cost of the car is five times the cost of the motorbike. Thus, in certain situations, comparison by division makes better sense than comparison by taking the difference. The comparison by division is the Ratio. In the next section, we shall learn more about ‘Ratios’.

12.2 Ratio

Consider the following:

Isha’s weight is 25 kg and her father’s weight is 75 kg. How many times Father’s weight is of Isha’s weight? It is three times.

Cost of a pen is ₹ 10 and cost of a pencil is ₹ 2. How many times the cost of a pen that of a pencil? Obviously it is five times.

In the above examples, we compared the two quantities in terms of ‘how many times’. This comparison is known as the Ratio. We denote ratio using symbol ‘:’

Consider the earlier examples again. We can say,

The ratio of father’s weight to Isha’s weight = $\frac{75}{25} = \frac{3}{1} = 3:1$

The ratio of the cost of a pen to the cost of a pencil = $\frac{10}{2} = \frac{5}{1} = 5:1$

Let us look at this problem.

In a class, there are 20 boys and 40 girls. What is the ratio of

(a) Number of girls to the total number of students.

(b) Number of boys to the total number of students.

Try These

1. In a class, there are 20 boys and 40 girls. What is the ratio of the number of boys to the number of girls?
2. Ravi walks 6 km in an hour while Roshan walks 4 km in an hour. What is the ratio of the distance covered by Ravi to the distance covered by Roshan?

First we need to find the total number of students, which is,

Number of girls + Number of boys = 20 + 40 = 60.

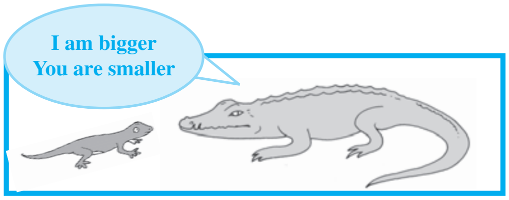
Then, the ratio of number of girls to the total number of students is $\frac{40}{60} = \frac{2}{3} = 2:3$

Find the answer of part (b) in the similar manner.

Now consider the following example.

Length of a house lizard is 20 cm and the length of a crocodile is 4 m.

“I am 5 times bigger than you”, says the lizard. As we can see this



is really absurd. A lizard's length cannot be 5 times of the length of a crocodile. So, what is wrong? Observe that the length of the lizard is in centimetres and length of the crocodile is in metres. So, we have to convert their lengths into the same unit.

Length of the crocodile = 4 m = 4 × 100 = 400 cm.

Therefore, ratio of the length of the crocodile to the length of the lizard

$= \frac{400}{20} = \frac{20}{1} = 20:1$.

Two quantities can be compared only if they are in the same unit.

Now what is the ratio of the length of the lizard to the length of the crocodile?

It is $\frac{20}{400} = \frac{1}{20} = 1:20$.

Observe that the two ratios 1 : 20 and 20 : 1 are different from each other. The ratio 1 : 20 is the ratio of the length of the lizard to the length of the crocodile whereas, 20 : 1 is the ratio of the length of the crocodile to the length of the lizard.

Now consider another example.

Length of a pencil is 18 cm and its diameter is 8 mm. What is the ratio of the diameter of the pencil to that of its length? Since the length and the diameter of the pencil are given in different units, we first need to convert them into same unit.

Thus, length of the pencil = 18 cm = 18 × 10 mm = 180 mm.

The ratio of the diameter of the pencil to that of the length of the pencil $= \frac{8}{180} = \frac{2}{45} = 2:45$.

Try These

1. Saurabh takes 15 minutes to reach school from his house and Sachin takes one hour to reach school from his house. Find the ratio of the time taken by Saurabh to the time taken by Sachin.
2. Cost of a toffee is 50 paise and cost of a chocolate is ₹ 10. Find the ratio of the cost of a toffee to the cost of a chocolate.
3. In a school, there were 73 holidays in one year. What is the ratio of the number of holidays to the number of days in one year?

A B

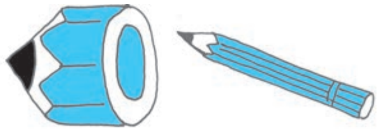
Think of some more situations where you compare two quantities of same type in different units.

We use the concept of ratio in many situations of our daily life without realising that we do so.

Compare the drawings A and B. B looks more natural than A. Why?

The legs in the picture A are too long in comparison to the other body parts. This is because we normally expect a certain ratio of the length of legs to the length of whole body.

Compare the two pictures of a pencil. Is the first one looking like a full pencil? No.



Why not? The reason is that the thickness and the length of the pencil are not in the correct ratio.

Same ratio in different situations :

Consider the following :

- Length of a room is 30 m and its breadth is 20 m. So, the ratio of length of the room to the breadth of the room = $\frac{30}{20} = \frac{3}{2} = 3:2$
- There are 24 girls and 16 boys going for a picnic. Ratio of the number of girls to the number of boys = $\frac{24}{16} = \frac{3}{2} = 3:2$

  The ratio in both the examples is 3 : 2.
- Note the ratios 30 : 20 and 24 : 16 in lowest form are same as 3 : 2. These are equivalent ratios.
- Can you think of some more examples having the ratio 3 : 2?

  It is fun to write situations that give rise to a certain ratio. For example, write situations that give the ratio 2 : 3.
- Ratio of the breadth of a table to the length of the table is 2 : 3.
- Sheena has 2 marbles and her friend Shabnam has 3 marbles.

Then, the ratio of marbles that Sheena and Shabnam have is 2 : 3.

Can you write some more situations for this ratio? Give any ratio to your friends and ask them to frame situations.

Ravi and Rani started a business and invested money in the ratio 2 : 3. After one year the total profit was ₹ 4,00,000.

Ravi said "we would divide it equally", Rani said "I should get more as I have invested more".

It was then decided that profit will be divided in the ratio of their investment.

Here, the two terms of the ratio 2 : 3 are 2 and 3.

Sum of these terms = 2 + 3 = 5

What does this mean?

This means if the profit is ₹ 5 then Ravi should get ₹ 2 and Rani should get ₹ 3. Or, we can say that Ravi gets 2 parts and Rani gets 3 parts out of the 5 parts.

i.e., Ravi should get $\frac{2}{5}$ of the total profit and Rani should get $\frac{3}{5}$ of the total profit.

If the total profit were ₹ 500

Ravi would get ₹ $\frac{2}{5} \times 500 =$ ₹ 200

and Rani would get $\frac{3}{5} \times 500 =$ ₹ 300

Now, if the profit were ₹ 4,00,000 could you find the share of each?

Ravi's share = ₹ $\frac{2}{5} \times 4,00,000$ = ₹ 1,60,000

And Rani's share = ₹ $\frac{3}{5} \times 4,00,000$ = ₹ 2,40,000

Can you think of some more examples where you have to divide a number of things in some ratio? Frame three such examples and ask your friends to solve them.

Let us look at the kind of problems we have solved so far.

Try These

1. Find the ratio of number of notebooks to the number of books in your bag.
2. Find the ratio of number of desks and chairs in your classroom.
3. Find the number of students above twelve years of age in your class. Then, find the ratio of number of students with age above twelve years and the remaining students.
4. Find the ratio of number of doors and the number of windows in your classroom.
5. Draw any rectangle and find the ratio of its length to its breadth.

Example 1 : Length and breadth of a rectangular field are 50 m and 15 m respectively. Find the ratio of the length to the breadth of the field.

Solution : Length of the rectangular field = 50 m

Breadth of the rectangular field = 15 m

The ratio of the length to the breadth is 50 : 15

The ratio can be written as $\frac{50}{15} = \frac{50 \div 5}{15 \div 5} = \frac{10}{3} = 10 : 3$

Thus, the required ratio is 10 : 3.

Rationalised 2023-24

Example 2 : Find the ratio of 90 cm to 1.5 m.

Solution : The two quantities are not in the same units. Therefore, we have to convert them into same units.

1.5 m = 1.5 × 100 cm = 150 cm.

Therefore, the required ratio is 90 : 150.

$$= \frac{90}{150} = \frac{90 \div 30}{150 \div 30} = \frac{3}{5}$$

Required ratio is 3 : 5.

Example 3 : There are 45 persons working in an office. If the number of females is 25 and the remaining are males, find the ratio of:

(a) The number of females to number of males.

(b) The number of males to number of females.

Solution : Number of females = 25

Total number of workers = 45

Number of males = 45 – 25 = 20

Therefore, the ratio of number of females to the number of males

= 25 : 20 = 5 : 4

And the ratio of number of males to the number of females

= 20 : 25 = 4 : 5.

(Notice that there is a difference between the two ratios 5 : 4 and 4 : 5).

Example 4 : Give two equivalent ratios of 6 : 4.

Solution : Ratio $6 : 4 = \frac{6}{4} = \frac{6 \times 2}{4 \times 2} = \frac{12}{8}$.

Therefore, 12 : 8 is an equivalent ratio of 6 : 4

Similarly, the ratio $6 : 4 = \frac{6}{4} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2}$

So, 3:2 is another equivalent ratio of 6 : 4.

Therefore, we can get equivalent ratios by multiplying or dividing the numerator and denominator by the same number.

Write two more equivalent ratios of 6 : 4.

Example 5 : Fill in the missing numbers :

$$\frac{14}{21} = \frac{\Box}{3} = \frac{6}{\Box}$$

Solution : In order to get the first missing number, we consider the fact that 21 = 3 × 7. i.e. when we divide 21 by 7 we get 3. This indicates that to get the missing number of second ratio, 14 must also be divided by 7.

When we divide, we have, 14 ÷ 7 = 2

Hence, the second ratio is $\frac{2}{3}$.

Similarly, to get third ratio we multiply both terms of second ratio by 3. (Why?)

Hence, the third ratio is $\frac{6}{9}$

Therefore, $\frac{14}{21} = \frac{\boxed{2}}{3} = \frac{6}{\boxed{9}}$ [These are all equivalent ratios.]

Example 6 : Ratio of distance of the school from Mary's home to the distance of the school from John's home is 2 : 1.

(a) Who lives nearer to the school?

(b) Complete the following table which shows some possible distances that Mary and John could live from the school.

Distance from Mary's home to school (in km.)	10		4		
Distance from John's home to school (in km.)	5	4		3	1

(c) If the ratio of distance of Mary's home to the distance of Kalam's home from school is 1 : 2, then who lives nearer to the school?

Solution : (a) John lives nearer to the school (As the ratio is 2 : 1).

(b)

Distance from Mary's home to school (in km.)	10	8	4	6	2
Distance from John's home to school (in km.)	5	4	2	3	1

(c) Since the ratio is 1 : 2, so Mary lives nearer to the school.

Example 7 : Divide ₹ 60 in the ratio 1 : 2 between Kriti and Kiran.

Solution : The two parts are 1 and 2.

Therefore, sum of the parts = 1 + 2 = 3.

This means if there are ₹ 3, Kriti will get ₹ 1 and Kiran will get ₹ 2. Or, we can say that Kriti gets 1 part and Kiran gets 2 parts out of every 3 parts.

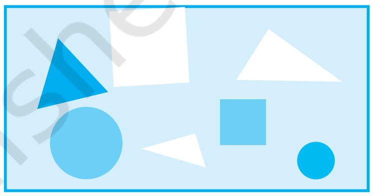
Therefore, Kriti's share = $\frac{1}{3} \times 60 =$ ₹ 20

And Kiran's share = $\frac{2}{3} \times 60 =$ ₹ 40.

EXERCISE 12.1

1. There are 20 girls and 15 boys in a class.
   (a) What is the ratio of number of girls to the number of boys?
   (b) What is the ratio of number of girls to the total number of students in the class?
2. Out of 30 students in a class, 6 like football, 12 like cricket and remaining like tennis. Find the ratio of
   (a) Number of students liking football to number of students liking tennis.
   (b) Number of students liking cricket to total number of students.

3. See the figure and find the ratio of
   (a) Number of triangles to the number of circles inside the rectangle.
   (b) Number of squares to all the figures inside the rectangle.
   (c) Number of circles to all the figures inside the rectangle.



4. Distances travelled by Hamid and Akhtar in an hour are 9 km and 12 km. Find the ratio of speed of Hamid to the speed of Akhtar.
5. Fill in the following blanks:

$$\frac{15}{18} = \frac{\square}{6} = \frac{10}{\square} = \frac{\square}{30}$$ [Are these equivalent ratios?]

6. Find the ratio of the following :
   (a) 81 to 108 (b) 98 to 63
   (c) 33 km to 121 km (d) 30 minutes to 45 minutes
7. Find the ratio of the following:
   (a) 30 minutes to 1.5 hours (b) 40 cm to 1.5 m
   (c) 55 paise to ₹ 1 (d) 500 mL to 2 litres
8. In a year, Seema earns ₹ 1,50,000 and saves ₹ 50,000. Find the ratio of
   (a) Money that Seema earns to the money she saves.
   (b) Money that she saves to the money she spends.
9. There are 102 teachers in a school of 3300 students. Find the ratio of the number of teachers to the number of students.
10. In a college, out of 4320 students, 2300 are girls. Find the ratio of
    (a) Number of girls to the total number of students.
    (b) Number of boys to the number of girls.

(c) Number of boys to the total number of students.

11. Out of 1800 students in a school, 750 opted basketball, 800 opted cricket and remaining opted table tennis. If a student can opt only one game, find the ratio of
    (a) Number of students who opted basketball to the number of students who opted table tennis.
    (b) Number of students who opted cricket to the number of students opting basketball.
    (c) Number of students who opted basketball to the total number of students.
12. Cost of a dozen pens is ₹ 180 and cost of 8 ball pens is ₹ 56. Find the ratio of the cost of a pen to the cost of a ball pen.
13. Consider the statement: Ratio of breadth and length of a hall is 2 : 5. Complete the following table that shows some possible breadths and lengths of the hall.
14. Divide 20 pens between Sheela and Sangeeta in the ratio of 3 : 2.

Breadth of the hall (in metres)	10		40
Length of the hall (in metres)	25	50	

15. Mother wants to divide ₹ 36 between her daughters Shreya and Bhoomika in the ratio of their ages. If age of Shreya is 15 years and age of Bhoomika is 12 years, find how much Shreya and Bhoomika will get.

16. Present age of father is 42 years and that of his son is 14 years. Find the ratio of
    (a) Present age of father to the present age of son.
    (b) Age of the father to the age of son, when son was 12 years old.
    (c) Age of father after 10 years to the age of son after 10 years.
    (d) Age of father to the age of son when father was 30 years old.

12.3 Proportion

Consider this situation :

Raju went to the market to purchase tomatoes. One shopkeeper tells him that the cost of tomatoes is ₹ 40 for 5 kg. Another shopkeeper gives the cost as 6 kg for ₹ 42. Now, what should Raju do? Should he purchase tomatoes from the first shopkeeper or from the second? Will the comparison by taking the difference help him decide? No. Why not?

Think of some way to help him. Discuss with your friends.

Consider another example.

Bhavika has 28 marbles and Vini has 180 flowers. They want to share these among themselves. Bhavika gave 14 marbles to Vini and Vini gave 90 flowers to Bhavika. But Vini was not satisfied. She felt that she had given more flowers to Bhavika than the marbles given by Bhavika to her.

What do you think? Is Vini correct?

To solve this problem both went to Vini's mother Pooja.

Pooja explained that out of 28 marbles, Bhavika gave 14 marbles to Vini.

Therefore, ratio is 14 : 28 = 1 : 2.

And out of 180 flowers, Vini had given 90 flowers to Bhavika.

Therefore, ratio is 90 : 180 = 1 : 2.

Since both the ratios are the same, so the distribution is fair.

Two friends Ashma and Pankhuri went to market to purchase hair clips. They purchased 20 hair clips for ₹ 30. Ashma gave ₹ 12 and Pankhuri gave ₹ 18. After they came back home, Ashma asked Pankhuri to give 10 hair clips to her. But Pankhuri said, "since I have given more money so I should get more clips. You should get 8 hair clips and I should get 12".

Can you tell who is correct, Ashma or Pankhuri? Why?

Ratio of money given by Ashma to the money given by Pankhuri

= ₹ 12 : ₹ 18 = 2 : 3

According to Ashma's suggestion, the ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 10 : 10 = 1 : 1

According to Pankhuri's suggestion, the ratio of the number of hair clips for Ashma to the number of hair clips for Pankhuri = 8 : 12 = 2 : 3

Now, notice that according to Ashma's distribution, ratio of hair clips and the ratio of money given by them is not the same. But according to the Pankhuri's distribution the two ratios are the same.

Hence, we can say that Pankhuri's distribution is correct.

Sharing a ratio means something!

Consider the following examples :

- Raj purchased 3 pens for ₹ 15 and Anu purchased 10 pens for ₹ 50. Whose pens are more expensive?

  Ratio of number of pens purchased by Raj to the number of pens purchased by Anu = 3 : 10.

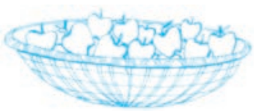
  Ratio of their costs = 15 : 50 = 3 : 10

  Both the ratios 3 : 10 and 15 : 50 are equal. Therefore, the pens were purchased for the same price by both.

- Rahim sells 2 kg of apples for ₹ 180 and Roshan sells 4 kg of apples for ₹ 360. Whose apples are more expensive?

  Ratio of the weight of apples = 2 kg : 4 kg = 1 : 2

  Ratio of their cost = ₹ 180 : ₹ 360 = 6 : 12 = 1 : 2



So, the ratio of weight of apples = ratio of their cost.

Since both the ratios are equal, hence, we say that they are in proportion. They are selling apples at the same rate.

If two ratios are equal, we say that they are in proportion and use the symbol '::' or '=' to equate the two ratios.

For the first example, we can say 3, 10, 15 and 50 are in proportion which is written as 3 : 10 :: 15 : 50 and is read as 3 is to 10 as 15 is to 50 or it is written as 3 : 10 = 15 : 50.

For the second example, we can say 2, 4, 180 and 360 are in proportion which is written as 2 : 4 :: 180 : 360 and is read as 2 is to 4 as 180 is to 360.

Let us consider another example.

A man travels 35 km in 2 hours. With the same speed would he be able to travel 70 km in 4 hours?

Now, ratio of the two distances travelled by the man is 35 to 70 = 1 : 2 and the ratio of the time taken to cover these distances is 2 to 4 = 1 : 2 .

Hence, the two ratios are equal i.e. 35 : 70 = 2 : 4.

Therefore, we can say that the four numbers 35, 70, 2 and 4 are in proportion.

Hence, we can write it as 35 : 70 :: 2 : 4 and read it as 35 is to 70 as 2 is to 4. Hence, he can travel 70 km in 4 hours with that speed.

Now, consider this example.

Cost of 2 kg of apples is ₹ 180 and a 5 kg watermelon costs ₹ 45.

Now, ratio of the weight of apples to the weight of watermelon is 2 : 5.

And ratio of the cost of apples to the cost of the watermelon is 180 : 45 = 4 : 1.

Here, the two ratios 2 : 5 and 180 : 45 are not equal, i.e. 2 : 5 ≠ 180 : 45

Therefore, the four quantities 2, 5, 180 and 45 are not in proportion.

Try These

Check whether the given ratios are equal, i.e. they are in proportion.

If yes, then write them in the proper form.

1. 1 : 5 and 3 : 15
2. 2 : 9 and 18 : 81
3. 15 : 45 and 5 : 25
4. 4 : 12 and 9 : 27
5. ₹ 10 to ₹ 15 and 4 to 6

If two ratios are not equal, then we say that they are not in proportion. In a statement of proportion, the four quantities involved when taken in order are known as respective terms. First and fourth terms are known as extreme terms. Second and third terms are known as middle terms.

For example, in 35 : 70 : : 2 : 4;

35, 70, 2, 4 are the four terms. 35 and 4 are the extreme terms. 70 and 2 are the middle terms.

Example 8 : Are the ratios 25g : 30g and 40 kg : 48 kg in proportion?

Solution : 25 g : 30 g = $\frac{25}{30}$ = 5 : 6

40 kg : 48 kg = $\frac{40}{48}$ = 5 : 6   So, 25 : 30 = 40 : 48.

Therefore, the ratios 25 g : 30 g and 40 kg : 48 kg are in proportion, i.e. 25 : 30 :: 40 : 48

The middle terms in this are 30, 40 and the extreme terms are 25, 48.

Example 9 : Are 30, 40, 45 and 60 in proportion?

Solution : Ratio of 30 to 40 = $\frac{30}{40}$ = 3 : 4.

Ratio of 45 to 60 = $\frac{45}{60}$ = 3 : 4.

Since, 30 : 40 = 45 : 60.

Therefore, 30, 40, 45, 60 are in proportion.

Example 10 : Do the ratios 15 cm to 2 m and 10 sec to 3 minutes form a proportion?

Solution : Ratio of 15 cm to 2 m = 15 : 2 × 100 (1 m = 100 cm)

= 3 : 40

Ratio of 10 sec to 3 min = 10 : 3 × 60 (1 min = 60 sec)

= 1 : 18

Since, 3 : 40 ≠ 1 : 18, therefore, the given ratios do not form a proportion.

EXERCISE 12.2

1. Determine if the following are in proportion.

   (a) 15, 45, 40, 120   (b) 33, 121, 9,96   (c) 24, 28, 36, 48

   (d) 32, 48, 70, 210   (e) 4, 6, 8, 12   (f) 33, 44, 75, 100

2. Write True (T) or False (F) against each of the following statements :

   (a) 16 : 24 :: 20 : 30   (b) 21: 6 :: 35 : 10   (c) 12 : 18 :: 28 : 12

(d) 8 : 9 :: 24 : 27 (e) 5.2 : 3.9 :: 3 : 4 (f) 0.9 : 0.36 :: 10 : 4

3. Are the following statements true?
   (a) 40 persons : 200 persons = ₹ 15 : ₹ 75
   (b) 7.5 litres : 15 litres = 5 kg : 10 kg
   (c) 99 kg : 45 kg = ₹ 44 : ₹ 20
   (d) 32 m : 64 m = 6 sec : 12 sec
   (e) 45 km : 60 km = 12 hours : 15 hours
4. Determine if the following ratios form a proportion. Also, write the middle terms and extreme terms where the ratios form a proportion.
   (a) 25 cm : 1 m and ₹ 40 : ₹ 160 (b) 39 litres : 65 litres and 6 bottles : 10 bottles
   (c) 2 kg : 80 kg and 25 g : 625 g (d) 200 mL : 2.5 litre and ₹ 4 : ₹ 50

12.4 Unitary Method

Consider the following situations:

- Two friends Reshma and Seema went to market to purchase notebooks. Reshma purchased 2 notebooks for ₹ 24. What is the price of one notebook?
- A scooter requires 2 litres of petrol to cover 80 km. How many litres of petrol is required to cover 1 km?

These are examples of the kind of situations that we face in our daily life. How would you solve these?

Reconsider the first example: Cost of 2 notebooks is ₹ 24.

Therefore, cost of 1 notebook = ₹ 24 ÷ 2 = ₹ 12.

Now, if you were asked to find cost of 5 such notebooks. It would be = ₹ 12 × 5 = ₹ 60

Reconsider the second example: We want to know how many litres are needed to travel 1 km.

For 80 km, petrol needed = 2 litres.

Therefore, to travel 1 km, petrol needed = $\frac{2}{80} = \frac{1}{40}$ litres.

Now, if you are asked to find how many litres of petrol are required to cover 120 km?

Then petrol needed = $\frac{1}{40} \times 120$ litres = 3 litres.

The method in which first we find the value of one unit and then the value of required number of units is known as Unitary Method.

Try These

1. Prepare five similar problems and ask your friends to solve them.
2. Read the table and fill in the boxes.

Time	Distance travelled by Karan	Distance travelled by Kriti
2 hours	8 km	6 km
1 hour	4 km	
4 hours		

We see that,

Distance travelled by Karan in 2 hours = 8 km

Distance travelled by Karan in 1 hour $= \frac{8}{2}$ km = 4 km

Therefore, distance travelled by Karan in 4 hours = $4 \times 4 = 16$ km

Similarly, to find the distance travelled by Kriti in 4 hours, first find the distance travelled by her in 1 hour.

Example 11 : If the cost of 6 cans of juice is ₹ 210, then what will be the cost of 4 cans of juice?

Solution : Cost of 6 cans of juice = ₹ 210

Therefore, cost of one can of juice = $\frac{210}{6}$ = ₹ 35

Therefore, cost of 4 cans of juice = ₹ 35×4 = ₹ 140.

Thus, cost of 4 cans of juice is ₹ 140.

Example 12 : A motorbike travels 220 km in 5 litres of petrol. How much distance will it cover in 1.5 litres of petrol?

Solution : In 5 litres of petrol, motorbike can travel 220 km.

Therefore, in 1 litre of petrol, motor bike travels = $\frac{220}{5}$ km

Therefore, in 1.5 litres, motorbike travels = $\frac{220}{5} \times 1.5$ km

$= \frac{220}{5} \times \frac{15}{10}$ km = 66 km.

Thus, the motorbike can travel 66 km in 1.5 litres of petrol.

Example 13 : If the cost of a dozen soaps is ₹ 153.60, what will be the cost of 15 such soaps?

Solution : We know that 1 dozen = 12

Since, cost of 12 soaps = ₹ 153.60

Therefore, cost of 1 soap = $\frac{153.60}{12}$ = ₹ 12.80

Therefore, cost of 15 soaps = ₹ 12.80 × 15 = ₹ 192

Thus, cost of 15 soaps is ₹ 192.

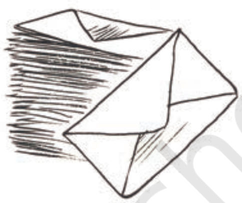
Example 14 : Cost of 105 envelopes is ₹ 350. How many envelopes can be purchased for ₹ 100?

Solution : In ₹ 350, the number of envelopes that can be purchased = 105

Therefore, in ₹ 1, number of envelopes that can be purchased = $\frac{105}{350}$

Therefore, in ₹ 100, the number of envelopes that can be

purchased = $\frac{105}{350} \times 100 = 30$



Thus, 30 envelopes can be purchased for ₹ 100.

Example 15 : A car travels 90 km in $2\frac{1}{2}$ hours.

(a) How much time is required to cover 30 km with the same speed?

(b) Find the distance covered in 2 hours with the same speed.

Solution : (a) In this case, time is unknown and distance is known. Therefore, we proceed as follows :

$2\frac{1}{2}$ hours = $\frac{5}{2}$ hours = $\frac{5}{2} \times 60$ minutes = 150 minutes.

90 km is covered in 150 minutes

Therefore, 1 km can be covered in $\frac{150}{90}$ minutes

Therefore, 30 km can be covered in $\frac{150}{90} \times 30$ minutes i.e. 50 minutes

Thus, 30 km can be covered in 50 minutes.

(b) In this case, distance is unknown and time is known. Therefore, we proceed as follows :

Distance covered in $2\frac{1}{2}$ hours (i.e. $\frac{5}{2}$ hours) = 90 km

Therefore, distance covered in 1 hour = $90 \div \frac{5}{2}$ km = $90 \times \frac{2}{5}$ = 36 km

Therefore, distance covered in 2 hours = 36 × 2 = 72 km.

Thus, in 2 hours, distance covered is 72 km.

EXERCISE 12.3

1. If the cost of 7 m of cloth is ₹ 1470, find the cost of 5 m of cloth.
2. Ekta earns ₹ 3000 in 10 days. How much will she earn in 30 days?
3. If it has rained 276 mm in the last 3 days, how many cm of rain will fall in one full week (7 days)? Assume that the rain continues to fall at the same rate.
4. Cost of 5 kg of wheat is ₹ 91.50.
   (a) What will be the cost of 8 kg of wheat?
   (b) What quantity of wheat can be purchased in ₹ 183?
5. The temperature dropped 15 degree celsius in the last 30 days. If the rate of temperature drop remains the same, how many degrees will the temperature drop in the next ten days?
6. Shaina pays ₹ 15000 as rent for 3 months. How much does she has to pay for a whole year, if the rent per month remains same?
7. Cost of 4 dozen bananas is ₹ 180. How many bananas can be purchased for ₹ 90?
8. The weight of 72 books is 9 kg. What is the weight of 40 such books?
9. A truck requires 108 litres of diesel for covering a distance of 594 km. How much diesel will be required by the truck to cover a distance of 1650 km?
10. Raju purchases 10 pens for ₹ 150 and Manish buys 7 pens for ₹ 84. Can you say who got the pens cheaper?
11. Anish made 42 runs in 6 overs and Anup made 63 runs in 7 overs. Who made more runs per over?

What have we discussed?

1. For comparing quantities of the same type, we commonly use the method of taking difference between the quantities.
2. In many situations, a more meaningful comparison between quantities is made by using division, i.e. by seeing how many times one quantity is to the other quantity. This method is known as comparison by ratio.

   For example, Isha's weight is 25 kg and her father's weight is 75 kg. We say that Isha's father's weight and Isha's weight are in the ratio 3 : 1.
3. For comparison by ratio, the two quantities must be in the same unit. If they are not, they should be expressed in the same unit before the ratio is taken.
4. The same ratio may occur in different situations.
5. Note that the ratio 3 : 2 is different from 2 : 3. Thus, the order in which quantities are taken to express their ratio is important.

6. A ratio may be treated as a fraction, thus the ratio 10 : 3 may be treated as $\frac{10}{3}$.

7. Two ratios are equivalent, if the fractions corresponding to them are equivalent. Thus, 3 : 2 is equivalent to 6 : 4 or 12 : 8.

8. A ratio can be expressed in its lowest form. For example, ratio 50 : 15 is treated as $\frac{50}{15}$; in its lowest form $\frac{50}{15} = \frac{10}{3}$. Hence, the lowest form of the ratio 50 : 15 is 10 : 3.

9. Four quantities are said to be in proportion, if the ratio of the first and the second quantities is equal to the ratio of the third and the fourth quantities. Thus, 3, 10, 15, 50 are in proportion, since $\frac{3}{10} = \frac{15}{50}$. We indicate the proportion by 3:10 :: 15:50, it is read as 3 is to 10 as 15 is to 50. In the above proportion, 3 and 50 are the extreme terms and 10 and 15 are the middle terms.

10. The order of terms in the proportion is important. 3, 10, 15 and 50 are in proportion, but 3, 10, 50 and 15 are not, since $\frac{3}{10}$ is not equal to $\frac{50}{15}$.

11. The method in which we first find the value of one unit and then the value of the required number of units is known as the unitary method. Suppose the cost of 6 cans is ₹ 210. To find the cost of 4 cans, using the unitary method, we first find the cost of 1 can. It is ₹ $\frac{210}{6}$ or ₹ 35. From this, we find the price of 4 cans as ₹ 35 × 4 or ₹ 140.

ANSWERS

EXERCISE 1.1

1. (a) Ten
(b) Ten
(c) Ten
(d) Ten
(e) Ten

2. (a) 73,75,307
(b) 9,05,00,041
(c) 7,52, 21,302
(d) 58,423,202
(e) 23,30,010

3. (a) 8,75,95,762 — Eight crore seventy-five lakh ninety-five thousand seven hundred sixty two.
(b) 85,46,283 — Eighty-five lakh forty-six thousand two hundred eighty-three.
(c) 9,99,00,046 — Nine crore ninety-nine lakh forty six.
(d) 9,84,32,701 — Nine crore eighty-four lakh, thirty-two thousand seven hundred one.

4. (a) 78,921,092 — Seventy-eight million, nine hundred twenty-one thousand, ninety-two.
(b) 7,452,283 — Seven million four hundred fifty-two thousand two hundred eighty-three.
(c) 99,985,102 — Ninety-nine million nine hundred eighty-five thousand, one hundred two.
(d) 48,049,831 — Forty-eight million forty-nine thousand eight hundred thirty one.

EXERCISE 1.2

1. 7,707 tickets
2. 3,020 runs
3. 2,28,800 votes
4. ₹ 6,86,659; second week, ₹ 1,14,877
5. 52,965
6. 87,575 screws
7. ₹ 30,592
8. 65,124
9. 18 shirts, 1 m 30 cm
10. 177 boxes
11. 22 km 500 m
12. 180 glasses.

EXERCISE 2.1

1. 11,000 ; 11,001 ; 11,002
2. 10,000 ; 9,999 ; 9,998
3. 0
4. 20
5. (a) 24,40,702 (b) 1,00,200 (c) 11,000,00 (d) 23,45,671
6. (a) 93 (b) 9,999 (c) 2,08,089 (e) 76,54,320
7. (a) 503 is on the left of 530 ; $503 < 530$
(b) 307 is on the left of 370 ; $307 < 370$
(c) 56,789 is on the left of 98,765 ; $56,789 < 98,765$
(d) 98,30,415 is on the left of 1,00,23,001 ; $98,30,415 < 1,00,23,001$
8. (a) F (b) F (c) T (d) T (e) T (f) F (g) F (h) F (i) T (j) F
(k) F (l) T (m) F

EXERCISE 3.1

1. (a) 1, 2, 3, 4, 6, 8, 12, 24 (b) 1, 3, 5, 15
(c) 1, 3, 7, 21 (d) 1, 3, 9, 27
(e) 1, 2, 3, 4, 6, 12 (f) 1, 2, 4, 5, 10, 20
(g) 1, 2, 3, 6, 9, 18 (h) 1, 23 (i) 1, 2, 3, 4, 6, 9, 12, 18, 36

2. (a) 5, 10, 15, 20, 25 (b) 8, 16, 24, 32, 40 (c) 9, 18, 27, 36, 45

3. (i) $\longrightarrow$ (b) (ii) $\longrightarrow$ (d) (iii) $\longrightarrow$ (a)
(iv) $\longrightarrow$ (f) (v) $\longrightarrow$ (e)

4. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99

EXERCISE 3.2

1. (a) even number (b) even number

2. (a) F (b) T (c) T (d) F
(e) F (f) F (g) F (h) T
(i) F (j) T

3. 17 and 71, 37 and 73, 79 and 97

4. Prime numbers : 2, 3, 5, 7, 11, 13, 17, 19
Composite numbers : 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 **5.** 7

6. (a) 3 + 41 (b) 5 + 31 (c) 5 + 19 (d) 5 + 13
(This could be one of the ways. There can be other ways also.)

7. 3, 5; 5, 7 ; 11, 13

8. (a) and (c) **9.** 90, 91, 92 , 93, 94, 95, 96

10. (a) 3 + 5 + 13 (b) 3 + 5 + 23
(c) 13 + 17 + 23 (d) 7 + 13 + 41
(This could be one of the ways. There can be other ways also.)

11. 2, 3 ; 2, 13; 3, 17; 7, 13; 11, 19

12. (a) prime number (b) composite number
(c) prime number, composite number (d) 2 (e) 4 (f) 2

EXERCISE 3.3

1.

Number	Divisible by								
	2	**3**	**4**	**5**	**6**	**8**	**9**	**10**	**11**
990	Yes	Yes	No	Yes	Yes	No	Yes	Yes	Yes
1586	Yes	No	No	No	No	No	No	No	No
275	No	No	No	Yes	No	No	No	No	Yes
6686	Yes	No	No	No	No	No	No	No	No
639210	Yes	Yes	No	Yes	Yes	No	No	Yes	Yes
429714	Yes	Yes	No	No	Yes	No	Yes	No	No
2856	Yes	Yes	Yes	No	Yes	Yes	No	No	No
3060	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No
406839	No	Yes	No	No	No	No	No	No	No

2. Divisible by 4 : (a) , (b), (c), (d), (f), (g), (h), (i)
Divisible by 8 : (b), (d), (f), (h)

3. (a), (f), (g), (i) **4.** (a), (b), (d), (e), (f)

5. (a) 2 and 8 (b) 0 and 9 **6.** (a) 8 (b) 6

EXERCISE 3.4

1. (a) 1, 2, 4 (b) 1, 5 (c) 1, 5 (d) 1, 2, 4, 8

2. (a) 1, 2, 4 (b) 1, 5

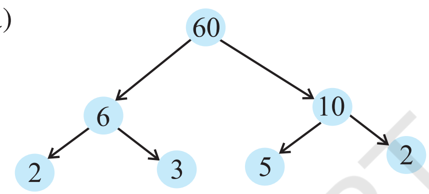
3. (a) 24, 48, 72 (b) 36, 72, 108

4. 12, 24, 36, 48, 60, 72, 84, 96

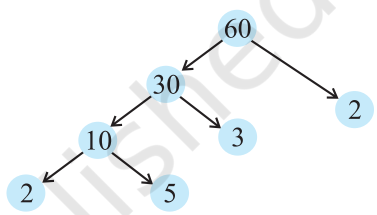
5. (a), (b), (e), (f) **6.** 60 **7.** 1, 2, 3, 4, 6

EXERCISE 3.5

1. (a)



(b)



2. 1 and the number itself

3. 9999, $9999 = 3 \times 3 \times 11 \times 101$

4. 10000, $10000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

5. $1729 = 7 \times 13 \times 19$
The difference of two consecutive prime factors is 6

6. (i) $2 \times 3 \times 4 = 24$ is divisible by 6.
(ii) $5 \times 6 \times 7 = 210$ is divisible by 6.

7. (b), (c)

8. No. Number 12 is divisible by both 4 and 6; but 12 is not divisible by 24.

9. $2 \times 3 \times 5 \times 7 = 210$

EXERCISE 3.6

1. (a) 6 (b) 6 (c) 6 (d) 9 (e) 12 (f) 34 (g) 35 (h) 7
(i) 9 (j) 3

2. (a) 1 (b) 2 (c) 1

3. No ; 1

EXERCISE 3.7

1. 3 kg **2.** 6930 cm **3.** 75 cm **4.** 120

5. 960 **6.** 7 minutes 12 seconds past 7 a.m.

7. 31 litres **8.** 95 **9.** 1152

10. (a) 36 (b) 60 (c) 30 (d) 60
Here, in each case LCM is a multiple of 3
Yes, in each case LCM = the product of two numbers

11. (a) 20 (b) 18 (c) 48 (d) 45
The LCM of the given numbers in each case is the larger of the two numbers.

EXERCISE 4.1

1. (a) O, B, C, D, E.

(b) Many answers are possible. Some are: $\overleftrightarrow{DE}, \overleftrightarrow{DO}, \overleftrightarrow{DB}, \overleftrightarrow{EO}$ etc.

(c) Many answers are possible. Some are: $\overrightarrow{DB}, \overrightarrow{DE}, \overrightarrow{OB}, \overrightarrow{OE}, \overrightarrow{EB}$ etc.

(d) Many answers are possible. Some are: $\overline{DE}, \overline{DO}, \overline{EO}, \overline{OB}, \overline{EB}$ etc.

2. $\overleftrightarrow{AB}, \overleftrightarrow{AC}, \overleftrightarrow{AD}, \overleftrightarrow{BA}, \overleftrightarrow{BC}, \overleftrightarrow{BD}, \overleftrightarrow{CA}, \overleftrightarrow{CB}, \overleftrightarrow{CD}, \overleftrightarrow{DA}, \overleftrightarrow{DB}, \overleftrightarrow{DC}$.

3. (a) Many answers. One answer is $\overleftrightarrow{AE}$.

(b) Many answers. One answer is $\overleftrightarrow{AE}$.

(c) $\overleftrightarrow{CO}$ or $\overleftrightarrow{OC}$

(d) Many answers are possible. Some are, $\overleftrightarrow{CO}, \overleftrightarrow{AE}$ and $\overleftrightarrow{AE}, \overleftrightarrow{EF}$.

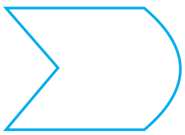
4. (a) Countless (b) Only one.

6. (a) T (b) T (c) T (d) F (e) F

(f) F (g) T (h) F (i) F (j) F (k) T

EXERCISE 4.2

1. Open : (a), (c); Closed : (b), (d), (e). **4.** (a) Yes (b) Yes

5. (a)  (b)  (c) Not possible.

EXERCISE 4.3

1. ∠A or ∠DAB; ∠B or ∠ABC; ∠C or ∠BCD; ∠D or ∠CDA

2. (a) A (b) A, C, D. (c) E, B, O, F.

EXERCISE 5.1

1. Chances of errors due to improper viewing are more.

2. Accurate measurement will be possible.

3. Yes. (because C is 'between' A and B).

4. B lies between A and C.

5. D is the mid point of $\overline{AG}$ (because, AD = DG = 3 units).

6. AB = BC and BC = CD, therefore, AB = CD

7. The sum of the lengths of any two sides of a triangle can never be less than the length of the third side.

EXERCISE 5.2

1. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$ (e) $\frac{3}{4}$ (f) $\frac{3}{4}$

2. (a) 6 (b) 8 (c) 8 (d) 2

3. (a) West (b) West (c) North (d) South

(To answer (d), it is immaterial whether we turn clockwise or anticlockwise, because one full revolution will bring us back to the original position).

4. (a) $\frac{3}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$

5. (a) 1 (b) 2 (c) 2 (d) 1 (e) 3 (f) 2

6. (a) 1 (b) 3 (c) 4 (d) 2 (clockwise or anticlockwise).

7. (a) 9 (b) 2 (c) 7 (d) 7

(We should consider only clockwise direction here).

EXERCISE 5.3

1. (i) $\to$ (c); (ii) $\to$ (d); (iii) $\to$ (a); (iv) $\to$ (e); (v) $\to$ (b).

2. Acute : (a) and(f); Obtuse : (b); Right: (c); Straight: (e); Reflex : (d).

EXERCISE 5.4

1. (i) 90°; (ii) 180°.

2. (a) T (b) F (c) T (d) T (e) T

3. (a) Acute: 23°, 89°; (b) Obtuse: 91°, 179°.

7. (a) acute (b) obtuse (if the angle is less than 180°)

(c) straight (d) acute (e) an obtuse angle.

9. 90°, 30°, 180°

10. The view through a magnifying glass will not change the angle measure.

EXERCISE 5.5

1. (a) and (c) **2.** 90°

3. One is a 30° – 60° – 90° set square; the other is a 45° – 45° – 90° set square. The angle of measure 90° (i.e. a right angle) is common between them.

4. (a) Yes (b) Yes (c) $\overline{BH}$, $\overline{DF}$ (d) All are true.

EXERCISE 5.6

1. (a) Scalene triangle (b) Scalene triangle (c) Equilateral triangle

(d) Right triangle (e) Isosceles right triangle (f) Acute-angled triangle

2. (i) $\to$ (e); (ii) $\to$ (g); (iii) $\to$ (a); (iv) $\to$ (f); (v) $\to$ (d);

(vi) $\to$ (c); (vii) $\to$ (b).

3. (a) Acute-angled and isosceles. (b) Right-angled and scalene.

(c) Obtuse-angled and isosceles. (d) Right-angled and isosceles.

(e) Equilateral and acute angled. (f) Obtuse-angled and scalene.

4. (b) is not possible. (Remember : The sum of the lengths of any two sides of a triangle has to be greater than the third side.)

EXERCISE 5.7

1. (a) T (b) T (c) T (d) T (e) F (f) F

2. (a) A rectangle with all sides equal becomes a square.

(b) A parallelogram with each angle a right angle becomes a rectangle.

(c) A rhombus with each angle a right angle becomes a square.

(d) All these are four-sided polygons made of line segments.

(e) The opposite sides of a square are parallel, so it is a parallelogram.

3. A square is a 'regular' quadrilateral

EXERCISE 5.8

1. (a) is not a closed figure and hence is not a polygon.

(b) is a polygon of six sides.

(c) and (d) are not polygons since they are not made of line segments.

2. (a) A Quadrilateral (b) A Triangle (c) A Pentagon (5-sided) (d) An Octagon

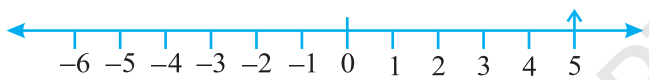
EXERCISE 6.1

1. (a) Decrease in weight (b) 30 km south (c) 80 m west

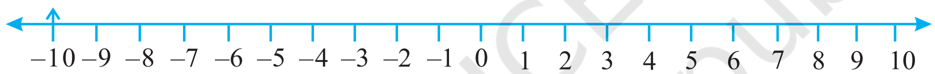
(d) Gain of ₹ 700 (e) 100 m below sea level

2. (a) + 2000 (b) – 800 (c) + 200 (d) – 700

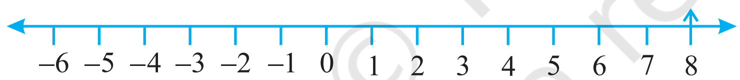
3. (a) + 5



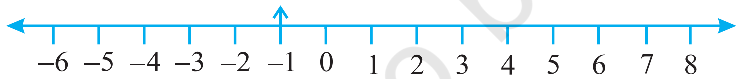
(b) – 10



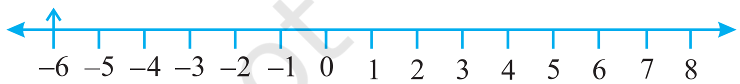
(c) + 8



(d) – 1



(e) – 6

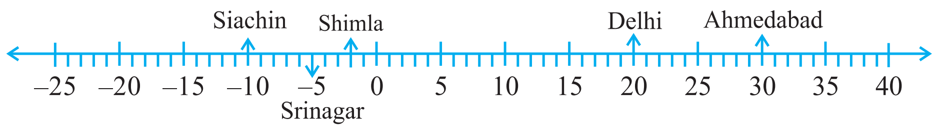


4. (a) F (b) negative integer (c) B → + 4, E → – 10

(d) E (e) D, C, B, A, O, H, G, F, E

5. (a) – 10°C, – 2°C, + 30°C, + 20°C, – 5°C

(b)



(c) Siachin (d) Ahmedabad and Delhi

6. (a) 9 (b) – 3 (c) 0 (d) 10 (e) 6 (f) 1

7. (a) – 6, – 5, – 4, – 3, – 2, – 1 (b) – 3,– 2, – 1, 0, 1, 2, 3
(c) – 14, – 13, – 12, – 11, – 10, – 9
(d) – 29, – 28, – 27, – 26, – 25, – 24

8. (a) – 19, – 18, – 17, – 16 (b) – 11, – 12, – 13, – 14

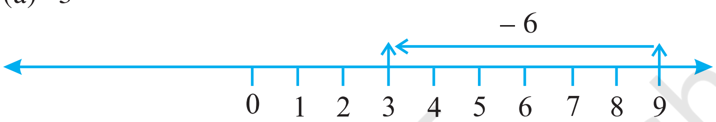
9. (a) T (b) F; – 100 is to the left of – 50 on number line
(c) F; greatest negative integer is – 1
(d) F; – 26 is smaller than – 25

10. (a) 2 (b) – 4 (c) to the left (d) to the right

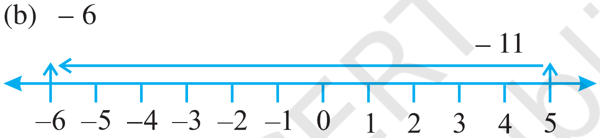
EXERCISE 6.2

1. (a) 8 (b) 0 (c) – 4 (d) – 5

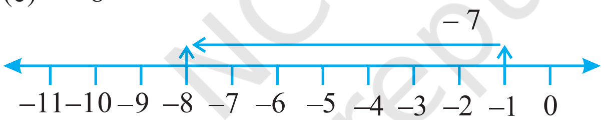
2. (a) 3



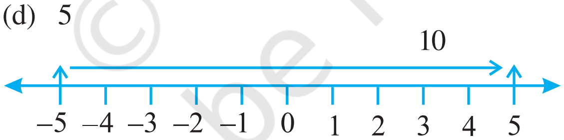
(b) – 6



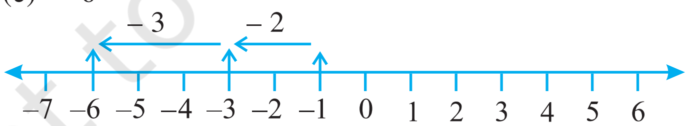
(c) – 8



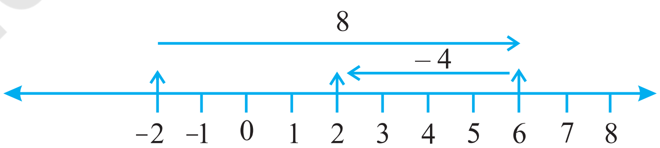
(d) 5



(e) – 6



(f) 2



3. (a) 4 (b) 5 (c) 9 (d) –100 (e) – 650 (f) – 317

4. (a) – 217 (b) 0 (c) – 81 (d) 50

5. (a) 4 (b) –38

EXERCISE 6.3

1. (a) 15 (b) – 18 (c) 3 (d) – 33 (e) 35 (f) 8

2. (a) < (b) > (c) > (d) >

3. (a) 8 (b) – 13 (c) 0 (d) – 8 (e) 5

4. (a) 10 (b) 10 (c) – 105 (d) 92

EXERCISE 7.1

1. (i) $\frac{2}{4}$ (ii) $\frac{8}{9}$ (iii) $\frac{4}{8}$ (iv) $\frac{1}{4}$ (v) $\frac{3}{7}$ (vi) $\frac{3}{12}$

(vii) $\frac{10}{10}$ (viii) $\frac{4}{9}$ (ix) $\frac{4}{8}$ (x) $\frac{1}{2}$

3. Shaded portions do not represent the given fractions.

4. $\frac{8}{24}$ **5.** $\frac{40}{60}$

6. (a) Arya will divide each sandwich into three equal parts, and give one part of each sandwich to each one of them.

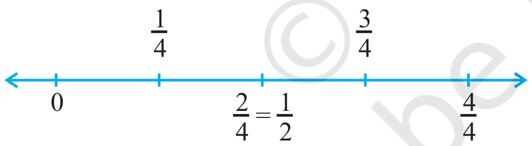
(b) $\frac{1}{3}$ **7.** $\frac{2}{3}$ **8.** 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12; $\frac{5}{11}$

9. 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113; $\frac{4}{12}$

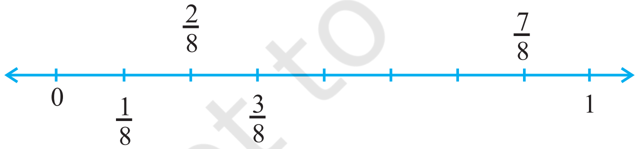
10. $\frac{4}{8}$ **11.** $\frac{3}{8}, \frac{5}{8}$

EXERCISE 7.2

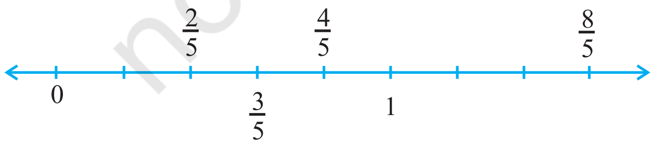
1. (a)



(b)



(c)



2. (a) $6\frac{2}{3}$ (b) $2\frac{1}{5}$ (c) $2\frac{3}{7}$ (d) $5\frac{3}{5}$ (e) $3\frac{1}{6}$ (f) $3\frac{8}{9}$

3. (a) $\frac{31}{4}$ (b) $\frac{41}{7}$ (c) $\frac{17}{6}$ (d) $\frac{53}{5}$ (e) $\frac{66}{7}$ (f) $\frac{76}{9}$

EXERCISE 7.3

1. (a) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$; Yes (b) $\frac{4}{12}, \frac{3}{9}, \frac{2}{6}, \frac{1}{3}, \frac{6}{15}$; No

2. (a) $\frac{1}{2}$ (b) $\frac{4}{6}$ (c) $\frac{3}{9}$ (d) $\frac{2}{8}$ (e) $\frac{3}{4}$ (i) $\frac{6}{18}$

(ii) $\frac{4}{8}$ (iii) $\frac{12}{16}$ (iv) $\frac{8}{12}$ (v) $\frac{4}{16}$

(a), (ii); (b), (iv); (c), (i); (d), (v); (e), (iii)

3. (a) 28 (b) 16 (c) 12 (d) 20 (e) 3

4. (a) $\frac{12}{20}$ (b) $\frac{9}{15}$ (c) $\frac{18}{30}$ (d) $\frac{27}{45}$

5. (a) $\frac{9}{12}$ (b) $\frac{3}{4}$

6. (a) equivalent (b) not equivalent (c) not equivalent

7. (a) $\frac{4}{5}$ (b) $\frac{5}{2}$ (c) $\frac{6}{7}$ (d) $\frac{3}{13}$ (e) $\frac{1}{4}$

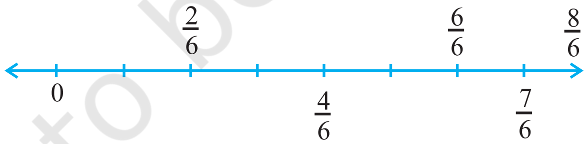
8. Ramesh $\rightarrow \frac{10}{20} = \frac{1}{2}$, Sheelu $\rightarrow \frac{25}{50} = \frac{1}{2}$, Jamaal $\rightarrow \frac{40}{80} = \frac{1}{2}$. Yes

9. (i) $\rightarrow$ (d) (ii) $\rightarrow$ (e) (iii) $\rightarrow$ (a) (iv) $\rightarrow$ (c) (v) $\rightarrow$ (b)

EXERCISE 7.4

1. (a) $\frac{1}{8} < \frac{3}{8} < \frac{4}{8} < \frac{6}{8}$ (b) $\frac{3}{9} < \frac{4}{9} < \frac{6}{9} < \frac{8}{9}$

(c)



$\frac{5}{6} > \frac{2}{6}, \frac{3}{6} > \frac{0}{6}, \frac{1}{6} < \frac{6}{6}, \frac{8}{6} > \frac{5}{6}$

2. (a) $\frac{3}{6} < \frac{5}{6}$ (b) $\frac{1}{7} < \frac{1}{4}$ (c) $\frac{4}{5} < \frac{5}{5}$ (d) $\frac{3}{5} > \frac{3}{7}$

4. (a) $\frac{1}{6} < \frac{1}{3}$ (b) $\frac{3}{4} > \frac{2}{6}$ (c) $\frac{2}{3} > \frac{2}{4}$ (d) $\frac{6}{6} = \frac{3}{3}$

(e) $\frac{5}{6} < \frac{5}{5}$

5. (a) $\frac{1}{2} > \frac{1}{5}$ (b) $\frac{2}{4} = \frac{3}{6}$ (c) $\frac{3}{5} < \frac{2}{3}$ (d) $\frac{3}{4} > \frac{2}{8}$

(e) $\frac{3}{5} < \frac{6}{5}$ (f) $\frac{7}{9} > \frac{3}{9}$ (g) $\frac{1}{4} = \frac{2}{8}$ (h) $\frac{6}{10} < \frac{4}{5}$

(i) $\frac{3}{4} < \frac{7}{8}$ (j) $\frac{6}{10} = \frac{3}{5}$ (k) $\frac{5}{7} = \frac{15}{21}$

6. (a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{4}{25}$ (d) $\frac{4}{25}$ (e) $\frac{1}{6}$ (f) $\frac{1}{5}$

(g) $\frac{1}{5}$ (h) $\frac{1}{6}$ (i) $\frac{4}{25}$ (j) $\frac{1}{6}$ (k) $\frac{1}{6}$ (l) $\frac{4}{25}$

(a), (e), (h), (j), (k) ; (b), (f), (g) ; (c), (d), (i), (l)

7. (a) No ; $\frac{5}{9} = \frac{25}{45}, \frac{4}{5} = \frac{36}{45}$ and $\frac{25}{45} \neq \frac{36}{45}$

(b) No ; $\frac{9}{16} = \frac{81}{144}, \frac{5}{9} = \frac{80}{144}$ and $\frac{81}{144} \neq \frac{80}{144}$ (c) Yes ; $\frac{4}{5} = \frac{16}{20}$

(d) No ; $\frac{1}{15} = \frac{2}{30}$ and $\frac{2}{30} \neq \frac{4}{30}$

8. Ila has read less **9.** Rohit

10. Same fraction ($\frac{4}{5}$) of students got first class in both the classes.

EXERCISE 7.5

1. (a) + (b) – (c) +

2. (a) $\frac{1}{9}$ (b) $\frac{11}{15}$ (c) $\frac{2}{7}$ (d) 1 (e) $\frac{1}{3}$

(f) 1 (g) $\frac{1}{3}$ (h) $\frac{1}{4}$ (i) $\frac{3}{5}$

3. The complete wall.

4. (a) $\frac{4}{10}$ ($= \frac{2}{5}$) (b) $\frac{8}{21}$ (c) $\frac{6}{6}$ (=1) (d) $\frac{7}{27}$

5. $\frac{2}{7}$

EXERCISE 7.6

1. (a) $\frac{17}{21}$ (b) $\frac{23}{30}$ (c) $\frac{46}{63}$ (d) $\frac{22}{21}$ (e) $\frac{17}{30}$

(f) $\frac{22}{15}$ (g) $\frac{5}{12}$ (h) $\frac{3}{6}\left(=\frac{1}{2}\right)$ (i) $\frac{23}{12}$ (j) $\frac{6}{6}$ (= 1) (k) 5

(l) $\frac{95}{12}$ (m) $\frac{9}{5}$ (n) $\frac{5}{6}$

2. $\frac{23}{20}$ metre **3.** $2\frac{5}{6}$

4. (a) $\frac{7}{8}$ (b) $\frac{7}{10}$ (c) $\frac{1}{3}$

5. (a)

$\frac{2}{3}$	$\frac{4}{3}$	2
$\frac{1}{3}$	$\frac{2}{3}$	1
$\frac{1}{3}$	$\frac{2}{3}$	1

(b)

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{5}{6}$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{7}{12}$
$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{4}$

6. Length of the other piece = $\frac{5}{8}$ metre

7. The distance walked by Nandini = $\frac{4}{10}\left(=\frac{2}{5}\right)$ km

8. Asha's bookshelf is more full; by $\frac{13}{30}$

9. Rahul takes less time; by $\frac{9}{20}$ minutes

EXERCISE 8.1

1. (a) 0.4 (b) 0.07 (c) 3 (d) 0.5 (e) 1.23
(f) 0.19 (g) both are same (h) 1.490 (i) both are same (j) 5.64

EXERCISE 8.2

1. (a) ₹ 0.05 (b) ₹ 0.75 (c) ₹ 0.20 (d) ₹ 50.90 (e) ₹ 7.25

2. (a) 0.15 m (b) 0.06 m (c) 2.45 m (d) 9.07 m (e) 4.19 m

3. (a) 0.5 cm (b) 6.0 cm (c) 16.4 cm (d) 9.8 cm (e) 9.3 cm

4. (a) 0.008 km (b) 0.088 km (c) 8.888 km (d) 70.005 km

5. (a) 0.002 kg (b) 0.1 kg (c) 3.750 kg (d) 5.008 kg (e) 26.05 kg

EXERCISE 8.3

1. (a) 38.587 (b) 29.432 (c) 27.63 (d) 38.355 (e) 13.175 (f) 343.89

2. ₹ 68.35 **3.** ₹ 26.30 **4.** 5.25 m

5. 3.042 km **6.** 22.775 km **7.** 18.270 kg

EXERCISE 8.4

1. (a) ₹ 2.50 (b) 47.46 m (c) ₹ 3.04 (d) 3.155 km (e) 1.793 kg

2. (a) 3.476 (b) 5.78 (c) 11.71 (d) 1.753

3. ₹ 14.35 **4.** ₹ 6.75 **5.** 15.55 m

6. 9.850 km **7.** 4.425 kg

EXERCISE 9.1

1.

Marks	Tally marks	Number of students
1	||	2
2	|||	3
3	|||	3
4	卌 ||	7
5	卌 |	6
6	卌 ||	7
7	卌	5
8	||||	4
9	|||	3

(a) 12 (b) 8

2.

Sweets	Tally marks	Number of students
Ladoo	卌 卌 |	11
Barfi	|||	3
Jalebi	卌 ||	7
Rasgulla	卌 ||||	9
		30

(b) Ladoo

3.

Numbers	Tally marks	How many times?
1	卌 ||	7
2	卌 |	6
3	卌	5
4	||||	4
5	卌 卌 |	11
6	卌 ||	7

(a) 4 (b) 5 (c) 1 and 6

4. (i) Village D (ii) Village C (iii) 3 (iv) 28

5. (a) VIII (b) No (c) 12

6. (a) Number of bulbs sold on Friday are 14. Similarly, number of bulbs sold on other days can be found.

(b) Maximum number of bulbs were sold on Sunday.

(c) Same number of bulbs were sold on Wednesday and Saturday.

(d) Minimum number of bulbs were sold on Wednesday and Saturday.

(e) 10 Cartons

7. (a) Martin (b) 700 (c) Anwar, Martin, Ranjit Singh

EXERCISE 10.1

1. (a) 12 cm (b) 133 cm (c) 60 cm (d) 20 cm (e) 15 cm
(f) 52 cm **2.** 100 cm or 1 m **3.** 7.5 m **4.** 106 cm

5. 9.6 km **6.** (a) 12 cm (b) 27 cm (c) 22 cm

7. 39 cm **8.** 48 m **9.** 5 m **10.** 20 cm

11. (a) 7.5 cm (b) 10 cm (c) 5 cm **12.** 10 cm

13. ₹ 20,000 **14.** ₹ 7200 **15.** Bulbul

16. (a) 100 cm (b) 100 cm (c) 100 cm (d) 100 cm
All the figures have same perimeter.

17. (a) 6 m (b) 10 m (c) Cross has greater perimeter

EXERCISE 10.2

1. (a) 9 sq units (b) 5 sq units (c) 4 sq units (d) 8 sq units (e) 10 sq units
(f) 4 sq units (g) 6 sq units (h) 5 sq units (i) 9 sq units (j) 4 sq units
(k) 5 sq units (l) 8 sq units (m) 14 sq units (n) 18 sq units

EXERCISE 10.3

1. (a) 12 sq cm (b) 252 sq cm (c) 6 sq km (d) 1.40 sq m

2. (a) 100 sq cm (b) 196 sq cm (c) 25 sq m

3. (c) largest area (b) smallest area

4. 6 m **5.** ₹ 8000 **6.** 3 sq m **7.** 14 sq m

8. 11 sq m **9.** 15 sq m

10. (a) 28 sq cm (b) 9 sq cm

11. (a) 40 sq cm (b) 245 sq cm (c) 9 sq cm

12. (a) 240 tiles (b) 42 tiles

EXERCISE 11.1

1. (a) $2n$ (b) $3n$ (c) $3n$ (d) $2n$ (e) $5n$
(f) $5n$ (g) $6n$

2. (a) and (d); The number of matchsticks required in each of them is 2

3. $5n$ **4.** $50b$ **5.** $5s$

6. t km **7.** $8r$, 64, 80 **8.** $(x - 4)$ years **9.** $l + 5$

10. $2x + 10$

11. (a) $3x + 1$, x = number of squares
(b) $2x + 1$, x = number of triangles

EXERCISE 12.1

1.	(a) 4 : 3	(b) 4 : 7		
2.	(a) 1 : 2	(b) 2 : 5		
3.	(a) 3 : 2	(b) 2 : 7	(c) 2 : 7	
4.	3 : 4	**5.** 5, 12, 25, Yes		
6.	(a) 3 : 4	(b) 14 : 9	(c) 3 : 11	(d) 2 : 3
7.	(a) 1 : 3	(b) 4 : 15	(c) 11 : 20	(d) 1 : 4
8.	(a) 3 : 1	(b) 1 : 2		
9.	17 : 550			
10.	(a) 115 : 216	(b) 101 : 115	(c) 101 : 216	
11.	(a) 3 : 1	(b) 16 : 15	(c) 5 : 12	
12.	15 : 7	**13.** 20 ; 100	**14.** 12 and 8	**15.** ₹ 20 and ₹ 16
16.	(a) 3 : 1	(b) 10 : 3	(c) 13 : 6	(d) 15 : 1

EXERCISE 12.2

1.	(a) Yes	(b) No	(c) No	(d) No	
	(e) Yes	(f) Yes			
2.	(a) T	(b) T	(c) F	(d) T	
	(e) F	(f) T			
3.	(a) T	(b) T	(c) T	(d) T	(e) F

4. (a) Yes, Middle Terms – 1 m, ₹ 40; Extreme Terms – 25 cm, ₹ 160

(b) Yes, Middle Terms – 65 litres, 6 bottles; Extreme Terms – 39 litres, 10 bottles

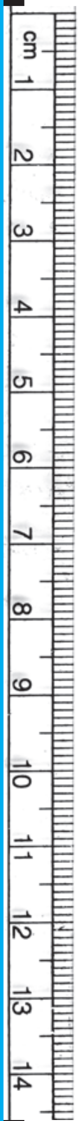
(c) No.

(d) Yes, Middle Terms – 2.5 litres, ₹ 4 ; Extreme Terms – 200 ml, ₹ 50

EXERCISE 12.3

1. ₹ 1,050	**2.** ₹ 9,000	**3.** 64.4 cm	
4. (a) ₹ 146.40	(b) 10 kg		
5. 5 degrees	**6.** ₹ 60, 000	**7.** 24 bananas	**8.** 5 kg
9. 300 litres	**10.** Manish	**11.** Anup	

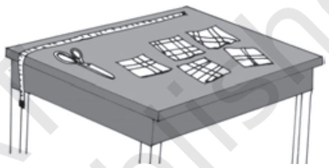
Note



BRAIN-TEASERS

1. From a basket of mangoes when counted in twos there was one extra, counted in threes there were two extra, counted in fours there were three extra, counted in fives there were four extra, counted in sixes there were five extra. But counted in sevens there were no extra. Atleast how many mangoes were there in the basket?

2. A boy was asked to find the LCM of 3, 5, 12 and another number. But while calculating, he wrote 21 instead of 12 and yet came with the correct answer. What could be the fourth number?



3. There were five pieces of cloth of lengths 15 m, 21 m, 36 m, 42 m, 48 m. But all of them could be measured in whole units of a measuring rod. What could be the largest length of the rod?

4. There are three cans. One of them holds exactly 10 litres of milk and is full. The other two cans can hold 7 litres and 3 litres respectively. There is no graduation mark on the cans. A customer asks for 5 litres of milk. How would you give him the amount he ask? He would not be satisfied by eye estimates.

5. Which two digit numbers when added to 27 get reversed?

6. Cement mortar was being prepared by mixing cement to sand in the ratio of 1:6 by volume. In a cement mortar of 42 units of volume, how much more cement needs to be added to enrich the mortar to the ratio 2:9?

7. In a solution of common salt in water, the ratio of salt to water was 30:70 as per weight. If we evaporate 100 grams of water from one kilogram of this solution, what will be the ratio of the salt to water by weight?

8. Half a swarm of bees went to collect honey from a mustard field. Three fourth of the rest went to a rose garden. The rest ten were still undecided. How many bees were there in all?

9. Fifteen children are sitting in a circle. They are asked to pass a handkerchief to the child next to the child immediately after them.

   The game stops once the handkerchief returns to the child it started from. This can be written as follows : 1→3→5→7→9→11→13→15→2→4→6→8 → 10→12 →14→1. Here, we see that every child gets the handkerchief.

(i) What would happen if the handkerchief were passed to the left leaving two children in between? Would every child get the handkerchief?

(ii) What if we leave three children in between? What do you see?

In which cases every child gets the handkerchief and in which cases not?

Try the same game with 16, 17, 18, 19, 20 children. What do you see?

10. Take two numbers 9 and 16. Divide 9 by 16 to get the remainder. What is the remainder when 2 × 9 is divided by 16, 3 × 9 divided by 16, 4 × 9 divided by 16, 5 × 9 divided by 16... 15 × 9 divided by 16. List the remainders. Take the numbers 12 and 14. List the remainders of 12, 12 × 2, 12 × 3, 12 × 4, 12 × 5, 12 × 6, 12 × 7, 12 × 8, 12 × 9, 12 × 10, 12 × 11, 12 × 12, 12 × 13 when divided by 14. Do you see any difference between above two cases?

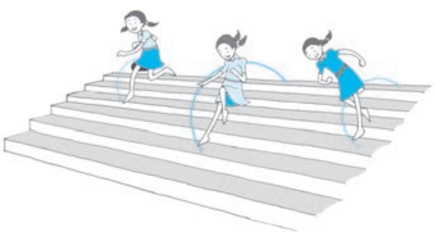
11. You have been given two cans with capacities 9 and 5 litres respectively. There is no graduation marks on the cans nor is eye estimation possible. How can you collect 3 litres of water from a tap? (You are allowed to pour out water from the can). If the cans had capacities 8 and 6 litres respectively, could you collect 5 litres?

12. The area of the east wall of an auditorium is 108 sq m, the area of the north wall is 135 sq m and the area of the floor is 180 sq m. Find the height of the auditorium.

13. If we subtract 4 from the digit at the units place of a two digit number and add 4 to the digit at the tens place then the resulting number is doubled. Find the number.

14. Two boatmen start simultaneously from the opposite shores of a river and they cross each other after 45 minutes of their starting from the respective shores. They rowed till they reached the opposite shore and returned immediately after reaching the shores. When will they cross each other again?

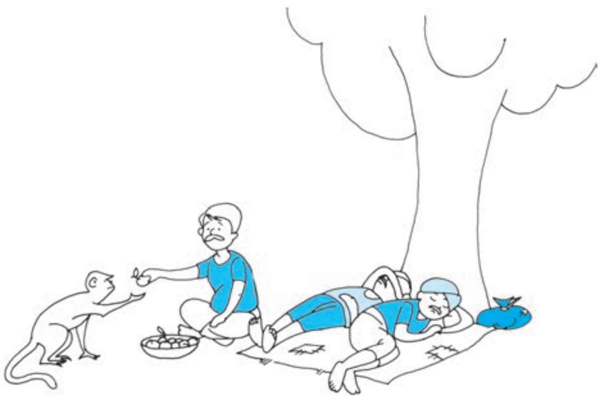
15. Three girls are climbing down a staircase. One girl climbs down two steps at one go. The second girl three steps at one go and the third climbs down four steps. They started together from the beginning of the staircase



leaving their foot marks. They all came down in complete steps and had their foot marks together at the bottom of the staircase. In how many steps would there be only one pair of foot mark?

Are there any steps on which there would be no foot marks.

16. A group of soldiers was asked to fall in line making rows of three. It was found that there was one soldier extra. Then they were asked to stand in rows of five. It was found there were left 2 soldiers. They were asked to stand in rows of seven. Then there were three soldiers who could not be adjusted. At least how many soldiers were there in the group?
17. Get 100 using four 9's and some of the symbols like +, –, ×, ¸ etc.
18. How many digits would be in the product $2 \times 2 \times 2 \ldots\ldots \times 2$ (30 times)?
19. A man would be 5 minutes late to reach his destination if he rides his bike at 30 km. per hour. But he would be 10 minutes early if he rides at the speed of 40 km per hour. What is the distance of his destination from where he starts?
20. The ratio of speeds of two vehicles is 2:3. If the first vehicle covers 50 km in 3 hours, what distance would the second vehicle covers in 2 hours?
21. The ratio of income to expenditure of Mr. Natarajan is 7:5. If he saves ₹ 2000 a month, what could be his income?
22. The ratio of the length to breadth of a lawn is 3:5. It costs ₹ 3200 to fence it at the rate of ₹ 2 a metre. What would be the cost of developing the lawn at the rate of ₹10 per square metre.
23. If one counts one for the thumb, two for the index finger, three for the middle finger, four for the ring finger, five for the little finger and continues counting backwards, six for the ring finger, seven for the middle finger, eight for the index finger, 9 for the thumb, ten for the index finger, eleven for the middle finger, twelve for the ring finger, thirteen for the little finger, fourteen for the ring finger and so on. Which finger will be counted as one thousand?
24. Three friends plucked some mangoes from a mango grove and collected them together in a pile and took nap after that. After some time, one of the friends woke up and divided the mangoes into three equal numbers. There was one



mango extra. He gave it to a monkey nearby, took one part for himself and slept again. Next the second friend got up unaware of what has happened, divided the rest of the mangoes into three equal shares. There was an extra mango. He gave it to the monkey, took one share for himself and slept again. Next the third friend got up not knowing what happened and divided the mangoes into three equal shares. There was an extra mango. He gave it to the monkey, took one share for himself and went to sleep again. After some time, all of them got up together to find 30 mangoes. How many mangoes did the friends pluck initially?

25. **The peculiar number**

    There is a number which is very peculiar. This number is three times the sum of its digits. Can you find the number?

26. Ten saplings are to be planted in straight lines in such way that each line has exactly four of them.

27. What will be the next number in the sequence?
    (a) 1, 5, 9, 13, 17, 21, ...
    (b) 2, 7, 12, 17, 22, ...
    (c) 2, 6, 12, 20, 30, ...
    (d) 1, 2, 3, 5, 8, 13, ...
    (e) 1, 3, 6, 10, 15, ...

28. Observe the pattern in the following statement:

    $31 \times 39 = 13 \times 93$

    The two numbers on each side are co-prime and are obtained by **reversing the digits** of respective numbers. Try to write some more pairs of such numbers.

ANSWERS

1. 119
2. 28
3. 3 m
4. The man takes an empty vessel other than these.

   With the help of 3 litres can he takes out 9 litres of milk from the 10 litres can and pours it in the extra can. So, 1 litre milk remains in the 10 litres can. With the help of 7 litres can he takes out 7 litres of milk from the extra can and pours it in the 10 litres can. The 10 litres can now has $1 + 7 = 8$ litres of milk.

With the help of 3 litres can he takes out 3 litres milk from the 10 litres can. The 10 litres can now has 8 – 3 = 5 litres of milk, which he gives to the customer.

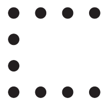
5. 14, 25, 36, 47, 58, 69
6. 2 units
7. 1 : 2
8. 80
9. (i) No, all children would not get it.

   (ii) All would get it.
10. 9, 2, 11, 4, 13, 6, 15, 8, 1, 10, 3, 12, 5, 14, 7.

    12, 10, 8, 6, 4, 2, 0, 12, 10, 8, 6, 4.
11. Fill the 9 litres can. Remove 5 litres from it using the 5 litres can. Empty the 5 litres can. Pour 4 litres remaining in the 9 litres can to the 5 litres can.

    Fill the 9 litres can again. Fill the remaining 5 litres can from the water in it. This leaves 8 litres in the 9 litres can. Empty the 5 litres can. Fill it from the 9 litres can. You now have 3 litres left in the 9 litres can.
12. Height = 9m
13. 36
14. 90 minutes
15. Steps with one pair of foot marks – 2, 3, 9, 10

    Steps with no foot marks – 1, 5, 7, 11
16. 52
17. $99 + \frac{9}{9}$
18. 10
19. 30 km
20. 50 km
21. ₹ 7000 per month

22. ₹ 15,00,000
23. Index finger
24. 106 mangoes
25. 27
26. One arrangement could be 
27. (a) 25 (b) 27 (c) 42 (d) 21 (e) 21
28. One such pair is $13 \times 62 = 31 \times 26$.